The relativistic self-energy in nuclear dynamics

Oliver Plohl

Institut für Theoretische Physik

Eberhard Karls Universität Tübingen

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Outline

Motivation & introduction

Self-energy in nuclear matter
   NN interaction models
   Restoration of symmetry of Lorentz group
   Results for NN potentials
   Self-energy from chiral EFT

Chiral condensate in nuclear matter
   In-Medium QCD sum rules
   Chiral condensate and effective nucleon mass

Summary & conclusions
Relativity in nuclear systems?

Relevance of relativity:

\[ \frac{k_F}{M} \simeq 1/4 \rightarrow \text{velocity } v \simeq 1/4c \]

→ moderate corrections from relativistic kinematics

But:

- Relativistic dynamics
  RMF, Hadronic many-body theory (DBHF), QCD sum rules

  \[ \rightarrow \Sigma_s \simeq -350 \text{ MeV}, \Sigma_0 \simeq +300 \text{ MeV} \]

- Cancellation in mean field potential \( U_{s.p.} \simeq \Sigma_0 + \Sigma_s \simeq -50 \text{ MeV} \)

- Large spin-orbit force \( U_{S.O.} \propto (\Sigma_0 - \Sigma_s) \vec{L} \cdot \vec{S} \simeq +750 \text{ MeV} \)

- Effective nucleon mass in nuclear matter \( M^* = M + \Sigma_s \)
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\[ \rightarrow \text{moderate corrections from relativistic kinematics} \]

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Summary & conclusions
Relativistic NN potentials

▶ Bonn A
field theoretical relativistic one-boson-exchange potential,
various scales and spin-isospin structure is associated with meson exchange,
long range part due to OPE, short/intermediate range = heavy mesons

▶ CD Bonn
charge independence breaking due to pion mass splitting,
readjustment of certain parameters in each partial wave (phenomenological
high-precision NN potential with 43 free parameters)
Non-relativistic NN potentials

- **Nijm 93 and Nijmegen I/II**
  long range part due to OPE, approximate OBE amplitudes

- **Argonne v_{18}**
  long range part due to OPE, intermediate and short range parametrized via operators \( O_\alpha \) and strength functions \( V_\alpha \)

- **Idaho potential**
  Chiral effective field theory, N^3LO, D. Entem and R. Machleidt, (29 free model parameters)

- **V_{lowk}**
  Derivation of an effective low-momentum potential \( V_{lowk} \) from modern NN potentials (out-integration of high-momentum modes, \( \Lambda \approx 2fm^{-1} \), and use of renormalization group methods)
Nucleon self-energy in Hartree-Fock approximation

\[ \Sigma = -i \int_F \left( \text{Tr}[GV] - GV \right) \]

- \(|LSJ\rangle \rightarrow \text{partial wave helicity basis} \rightarrow \text{plane wave helicity basis} \rightarrow \text{covariant operator basis}\)

- \(\text{translational and rotational invariance, parity conservation, time reversal invariance}\)

\[ \Sigma(k, k_F) = \Sigma_s(k, k_F) - \gamma_0 \Sigma_0(k, k_F) + \gamma \cdot k \Sigma_v(k, k_F) \]
Large scalar/vector fields

→ Mapping of NN potentials on covariant operator basis
→ large scalar/vector fields → universal feature of NN interaction

O.P., Fuchs, van Dalen, PRC 73 (2006) 014003
Role of contact terms

LO

\[ V = -\frac{g^{2}}{4\pi^{2}} \frac{\bar{\sigma}_{1} \cdot \vec{k} \bar{\sigma}_{2} \cdot \vec{k}}{q^{2} + m^{2}_{\pi}}, \quad V = C_{S} + C_{T} \bar{\sigma}_{1} \cdot \bar{\sigma}_{2} \]

NLO

leading order 2\pi exchange

\[ V = \ldots + C_{5}(-i \vec{S} \cdot (\vec{q} \times \vec{q}')) + \ldots + C_{7}(\ldots) \]

SO force (NLO contact terms) → large scalar/vector fields
→ Nucleon mass \( M^* = M + \Sigma_{S} \) → short-distance physics

O.P., C. Fuchs, PRC 74 (2006) 034325
Role of contact terms

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V = -\frac{g^2}{4f^2} \frac{\bar{\sigma}_1 \cdot \vec{k} \bar{\sigma}_2 \cdot \vec{k}}{q^2 + m_f^2}, \quad V = C_S + C_T \bar{\sigma}_1 \cdot \bar{\sigma}_2
\]

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leading order \(2\pi\) exchange

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O.P., C. Fuchs, PRC 74 (2006) 034325
Relativity in nuclear systems?

What’s known
Finite nuclei (RMF)
large scalar/vector fields $\Rightarrow$ SO force

What’s new
NN-scattering
large scalar/vector fields $\Leftarrow$ SO force
Relativity in nuclear systems?

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Summary & conclusions
Connection to QCD sum rules

\[ \Sigma_s = -\frac{8\pi^2}{\Lambda_B^2} [\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0] \]
\[ = -\frac{8\pi^2}{\Lambda_B^2} \frac{\sigma_N m_u + m_d}{\sigma_N M} \rho S \]
\[ = -\frac{\rho S}{m_\pi^2 f_\pi^2} \rho S \]
\[ -\Sigma_0 = -\frac{64\pi^2}{3\Lambda_B^2} \langle \bar{q} \gamma_0 q \rangle_\rho \]
\[ = -\frac{32\pi^2}{\Lambda_B^2} \rho \]

Joffe formulae

QCD sum rules and chiral EFT fields well comparable at moderate densities (both obtained to leading order in density)

O.P., C. Fuchs, PRC 74 (2006) 034325
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O.P., C. Fuchs, PRC 74 (2006) 034325
Nucleon mass in matter

\[ M^* = M + \Sigma_s \]

QCD sum rules

\[ \frac{M^*}{M} = 1 - \frac{\sigma_N}{m^2 f^2} \rho_B \]

Model independent prediction

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m^2 f^2} \rho_B \]

→ consistent comparison of effective nucleon mass and chiral condensate in matter

\[ \frac{M^*}{M} \equiv \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \]
Chiral condensate in nuclear matter

Order parameter of spontaneous chiral symmetry breaking
\[ \bar{q}q \equiv \frac{1}{2}(\bar{u}u + \bar{d}d), \quad m \equiv \frac{1}{2}(m_u + m_d) \]

\[ \langle \bar{q}q \rangle_0 = -(225 \pm 25 \text{ MeV})^3 \]

From Hellmann-Feynman theorem

\[ 2m (\langle \bar{q}q \rangle_{\rho_B} - \langle \bar{q}q \rangle_0) = m \frac{d\mathcal{E}}{dm} \]

Energy density
\[ \mathcal{E} = M\rho_B + \frac{E}{A}\rho_B \]

Equation-of-state
\[ \frac{E}{A} = \frac{1}{\rho} \int_F \frac{d^3k}{2\pi^3} \left[ \frac{k^2}{2M} + \frac{1}{2} U_{\text{s.p.}}(k, k_F) \right] \]
Chiral condensate in nuclear matter

\[
\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho_B}{m_\pi^2 f_\pi^2} \left[ \sigma_N + m \frac{d}{dm} \frac{E}{A} \right]
\]

Gell-Mann-Oakes-Renner \quad 2m\langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2,

pion-nucleon sigma-term \quad \sigma_N = m \frac{dM}{dm} = \langle N | m\bar{q}q | N \rangle

Comparison of \(\frac{M^*}{M}\) and \(\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_0}\) is done with
the same chiral EFT interaction and at the same order

Quark mass dependence of the nuclear forces,
Chiral condensate in nuclear matter at NLO

Leading order approximation
\[
\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \rho
\]

NLO chiral EFT potential (Hartree-Fock)
\[
\frac{M^*}{M} = 1 + \frac{\Sigma_S}{M}
\]

\[
\rightarrow \frac{M^*}{M} \neq \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0}
\]

\(M^*\) generated by NLO contact interactions

Change of chiral condensate mainly due to virtual low momentum pions
Motivation & introduction
Self-energy $\Sigma$
Chiral condensate
Summary

In-Medium QCD sum rules
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Chiral condensate in nuclear matter at NLO

Leading order approximation
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\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m^2_\pi f^2_\pi} \rho
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$M^*$ generated by NLO contact interactions
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Leading order approximation
\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m^2_\pi f^2_\pi} \rho \]

NLO chiral EFT potential (Hartree-Fock)
\[ \frac{M^*}{M} = 1 + \frac{\Sigma_S}{M} \]
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\( M^* \) generated by NLO contact interactions
Change of chiral condensate mainly due to virtual low momentum pions
Chiral condensate in nuclear matter

\[
\frac{\langle q\bar{q} \rangle_{\rho_N}}{\langle q\bar{q} \rangle_0} = \frac{\rho_N}{\rho_0}
\]

- short range NN correlations have minor influence on the density dependence of the chiral condensate
Summary & conclusions

- Relativistic self-energy
  → Vacuum structure of $NN$ interaction enforces the generation of large scalar/vector fields ($\approx 300 - 350$ MeV)

- Self-energy from chiral EFT
  NLO contact terms (LEC $C_5$ connected to SO Force)
  → large scalar/vector fields

- Scalar condensate in matter (NLO)
  Effective nucleon mass does not depend only on the scalar condensate (20%), different physical origin
Summary & conclusions

- Relativistic self-energy
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Thank you!
One-boson exchange potentials

**Bonn and CD-Bonn potentials**

\[ V(q', q) = \sum_{\alpha=s,ps,\nu} \tilde{V}_\alpha(q', q) \mathcal{F}_\alpha^2(q', q; \lambda_\alpha) \]

\[ -i\tilde{V}_\alpha(q', q) = \frac{\bar{u}(-q') \kappa_2^{(\alpha)} u(-q) P_\alpha \bar{u}(q') \kappa_1^{(\alpha)} u(q)}{(q' - q)^2 - m_\alpha^2}, \quad u_\lambda(q) = \sqrt{\frac{E + M}{2M}} \left( \frac{1}{2\lambda |q| E + M} \right) \chi_\lambda \]

**Dirac structure**

\[ \kappa^{(s)} = g_s 1, \quad \kappa^{(ps)} = g_{ps} \frac{q' - q}{2M} i\gamma_5, \quad \kappa^{(v)} = g_v \gamma^\mu + \frac{f_v}{2M} i\sigma^{\mu\nu} \]

→ long range = OPE, short/intermediate range = heavy mesons
Non-relativistic potentials

Low energy expansion of OBE potential

\[
V(q', q) = \sum_{\alpha=1,5} \left[ V_\alpha + V'_\alpha \tau_1 \cdot \tau_2 \right] O_\alpha
\]

\[
O_1 = 1,
\]

\[
O_2 = \sigma_1 \cdot \sigma_2,
\]

\[
O_3 = (\sigma_1 \cdot k)(\sigma_2 \cdot k),
\]

\[
O_4 = \frac{i}{2}(\sigma_1 + \sigma_2) \cdot n,
\]

\[
O_5 = (\sigma_1 \cdot n)(\sigma_2 \cdot n),
\]

\[
k = q' - q,
\]

\[
P = \frac{1}{2}(q' + q),
\]

\[
n = q \times q' \equiv P \times k,
\]
Nucleon self-energy in Hartree-Fock approximation

\[ \Sigma = -i \int_F \left( \text{Tr}[GV] - GV \right) \]

**Fermi covariants**

\[ S = 1 \otimes 1, \quad V = \gamma^\mu \otimes \gamma_\mu, \quad T = \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}, \quad P = \gamma_5 \otimes \gamma_5, \quad A = \gamma_5 \gamma^\mu \otimes \gamma_5 \gamma_\mu \]

**Pseudovector choice**

\[ \Gamma_m = \{ S, \tilde{S}, (A - \tilde{A}), PV, \tilde{PV} \} \]

**|LSJ⟩ → partial wave helicity basis → plane wave helicity basis → Covariant operator basis**

\[ \hat{V}^I(|q|, \theta) = g^I_S(|q|, \theta) S - g^I_{\tilde{S}}(|q|, \theta) \tilde{S} + g^I_A(|q|, \theta) (A - \tilde{A}) + g^I_{PV}(|q|, \theta) PV - g^I_{\tilde{PV}}(|q|, \theta) \tilde{PV} \]
Lorentz invariant amplitudes

- $g_S$
- $g_A$
- $g_{PV}$
- $g_{\sim}$

$q [\text{MeV}]$

$[\text{fm}^2]$
Chiral $NN$ potential

\[ \mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} \]

- **2N forces**
  - $Q^0_{LO}$
  - $Q^2_{NLO}$
  - $Q^3_{N^2LO}$
  - $Q^4_{N^3LO}$

- **3N forces**
  - $\ldots$

- **4N forces**
  - $\ldots$

\[ (\frac{Q}{\Lambda})^\nu \]

$Q$ is momentum (derivative) or pion mass $m_\pi$

$\Lambda$ is chiral symmetry breaking scale
Motivation & introduction
Self-energy $\Sigma$
Chiral condensate
Summary

Remarks

Single particle potential $U_{s.p.}$

$$U_{s.p.}(k) = \frac{M}{E_k} \langle \bar{u}(k)|\Sigma|u(k) \rangle$$

$$= M\Sigma_s/E_k - \Sigma_0 + \Sigma_v k^2/E_k$$

Oliver Plohl The relativistic self-energy in nuclear dynamics
Chiral condensate in nuclear matter

\[ \langle \psi(\lambda) | \frac{d}{d\lambda} H(\lambda) | \psi(\lambda) \rangle = \frac{d}{d\lambda} E(\lambda) \]

\[ \langle \psi(\lambda) | \frac{d}{d\lambda} H(\lambda) | \psi(\lambda) \rangle = \frac{d}{d\lambda} \langle \psi(\lambda) | H(\lambda) | \psi(\lambda) \rangle \]

Explicit chiral symmetry breaking \( \mathcal{H}_{QCD} = \mathcal{H}_0 + \mathcal{H}_m, \mathcal{H}_m = m_u \bar{u}u + m_d \bar{d}d + \cdots \)

Introduce \( \bar{q}q \equiv \frac{1}{2}(\bar{u}u + \bar{d}d), \quad m_q \equiv \frac{1}{2}(m_u + m_d), \quad \delta m_q = m_d - m_u \)

\[ \mathcal{H}_m = 2m_q \bar{q}q - \frac{1}{2} \delta m_q (\bar{u}u - \bar{d}d) + \cdots \]

Identify \( \lambda \rightarrow m_q \) and \( H \rightarrow \int d^3x \mathcal{H}_{QCD} \)

\[ 2m_q \langle \psi(m_q) | \int d^3x \bar{q}q | \psi(m_q) \rangle = m_q \frac{d}{dm_q} \langle \psi(m_q) | \int d^3x \mathcal{H}_{QCD} | \psi(m_q) \rangle. \]

\( |\psi(m_q)\rangle = |\rho_N\rangle \) (ground state of nuclear matter at \( \rho_N \))

\( |\psi(m_q)\rangle = |0\rangle \) (vacuum state)

Taking the difference of these two cases one obtains

\[ 2m_q (\langle \bar{q}q \rangle_{\rho_N} - \langle \bar{q}q \rangle_0) = m_q \frac{d}{dm_q} (\mathcal{E}(\rho_N) - \mathcal{E}(0)) . \]
Chiral condensate in nuclear matter

\[ 2m(\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0) = m \frac{d}{dm} \left( M + \frac{E}{A} \right) \rho \]

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 + \frac{m \rho}{2m \langle \bar{q}q \rangle_0} \frac{d}{dm} \left( M + \frac{E}{A} \right) \]

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{m^2 f^2_\pi} \left[ \sigma_N + m \frac{d}{dm} \frac{E}{A} \right] \]

Gell-Mann-Oakes-Renner (GOR) relation \[ 2m\langle \bar{q}q \rangle_0 = -m^2 f^2_\pi \],

pion-nucleon sigma-term \[ \sigma_N = m \frac{dM}{dm} = \langle N|m\bar{q}q|N \rangle \]
Chiral condensate in nuclear matter

DBHF with OBE Potential

\[
m \frac{dE}{dm} = \sum_{S, \nu, \pi, \rho} \left[ \frac{\partial E}{\partial m} \frac{dm_i}{dm} + \frac{\partial E}{\partial g_i} \frac{dg_i}{dm_i} + \cdots \right] + \sigma_N \frac{\partial E}{\partial M}
\]

\[
\sigma_S \equiv m \frac{dm_S}{dm} = C_S \sigma_N
\]

\[0.5 < C_S < 1\]


Fig. 3. The ratio \(\langle \bar{q} q \rangle_\rho / \langle \bar{q} q \rangle_0\) of the chiral condensate at baryon density \(\rho\) with respect to its value at \(\rho = 0\). Dashed curve: leading order result with \(\sigma_N = \sigma_N\). The solid curves correspond to different scalar “sigma terms” as in Fig. 2.
Chiral condensate in nuclear matter

\[
\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{m^2 f^2_\pi} \left[ \sigma_N + m \frac{d}{dm} \frac{E}{A} \right]
\]

\[
\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{m^2 f^2_\pi} \left[ \sigma_N + m \frac{\partial (E/A)}{\partial M} \frac{dM}{dm} + m \frac{\partial (E/A)}{\partial m_\pi} \frac{dm_\pi}{dm} + \cdots \right]
\]

\[
\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{\rho^X} \left[ 1 + \frac{\partial (E/A)}{\partial M} \frac{dM}{dm} + \frac{\partial (E/A)}{\partial m_\pi} \frac{m_\pi}{2\sigma_N} \right]
\]

\[
2m\langle \bar{q}q \rangle_0 = -m^2 f^2_\pi \quad \sigma_N = m \frac{dM}{dm} \quad dm_\pi = m_\pi \frac{dm}{2m} \quad \rho^X \equiv \frac{m^2 f^2_\pi}{\sigma_N}
\]

Quark mass dependence of the nuclear forces,