Double beta decay with Realistic Forces in deformed nuclei

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Collaboration with

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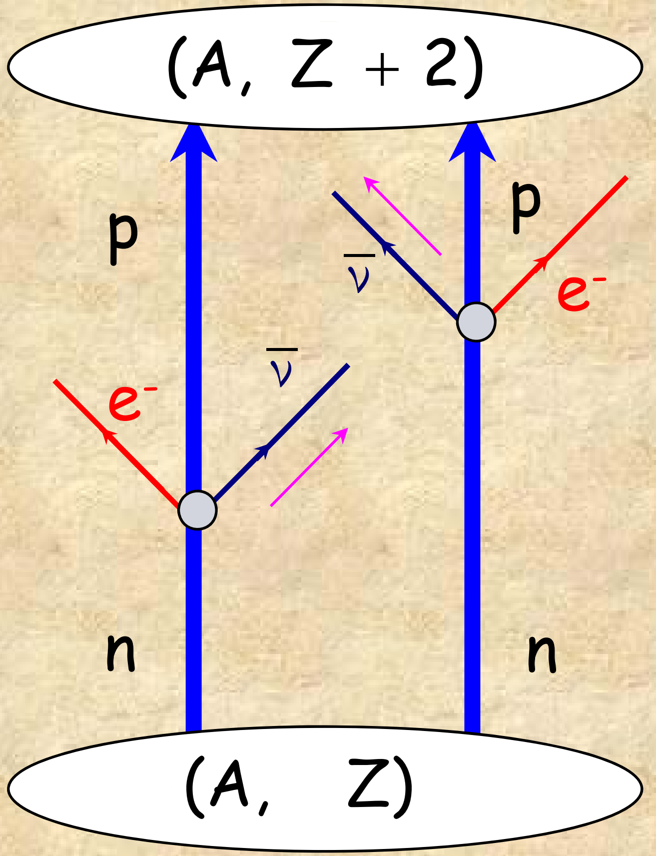
Two neutrino double beta decay (2νββ)

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Second order weak process within SM

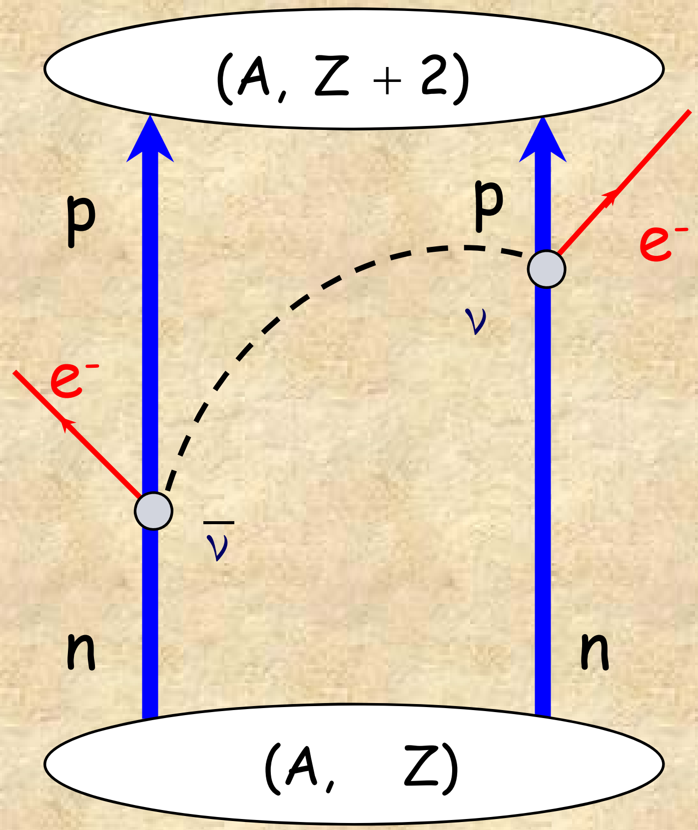


Neutrinoless double beta decay ($0\nu\beta\beta$)

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-}$$

Forbidden in SM electroweak interaction and it may occur if lepton number conservation is not an exact symmetry of nature .

If it is observed ⇨ **neutrino is massive Majorana particle**

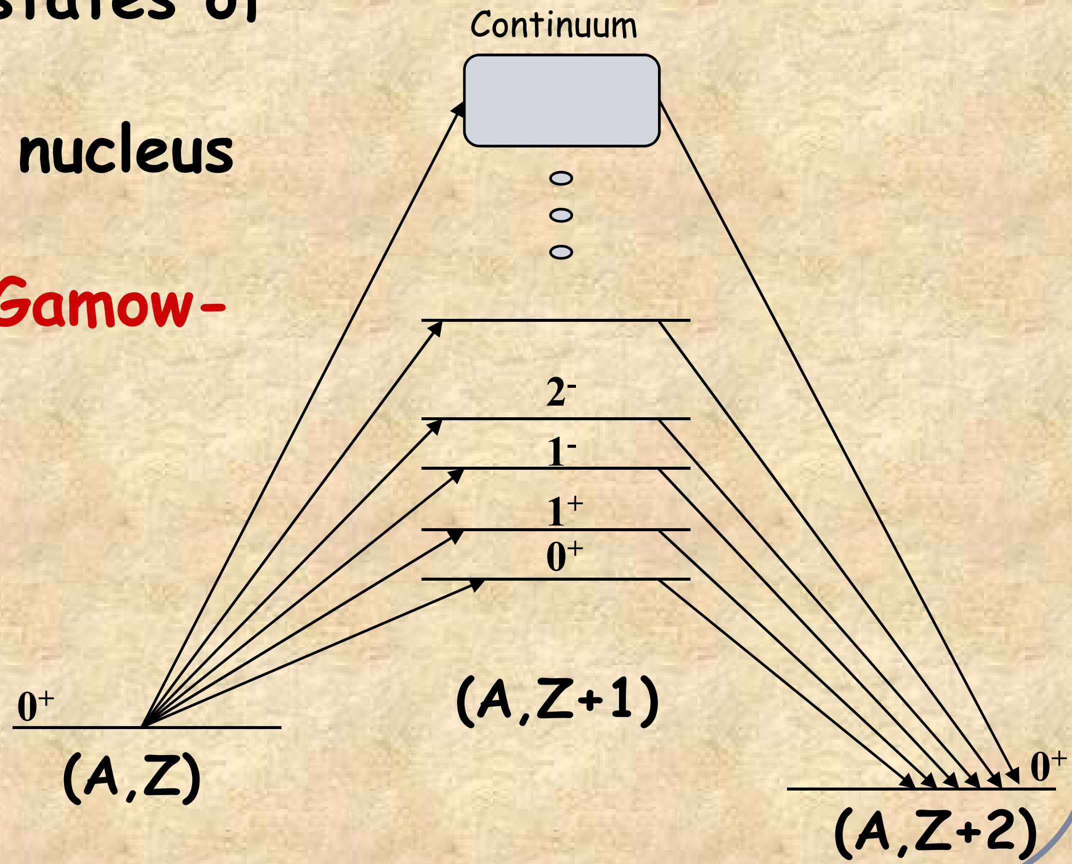


Light neutrino exchange mechanism

Nuclear Dynamics of ββ Decay

Virtual excitation of states of all multipolarities in intermediate (A,Z+1) nucleus

(2νββ only 1⁺ states ,Gamow-Teller transitions)



measured $T^{2\nu}_{1/2}$ (A. Barabash, 2005)

Isotope	$T^{2\nu}_{1/2}$, in 10^{19} y
^{48}Ca	$4.2^{+2.1}_{-1.0}$
^{76}Ge	150 ± 10
^{82}Se	9.2 ± 0.7
^{96}Zr	2.0 ± 0.3
^{100}Mo	0.71 ± 0.04
^{116}Cd	3.0 ± 0.2
^{128}Te	$(2.5 \pm 0.3) \times 10^5$
^{130}Te	90 ± 10
^{136}Xe	> 81 (90% CL)
^{150}Nd	0.78 ± 0.07
^{238}U	200 ± 60

Matrix Elements

The rate of $2\nu\beta\beta$ decay is

$$1/T_{1/2}^{2\nu} = G_{2\nu}\left|M_{2\nu}\right|^2$$

The rate of $0\nu\beta\beta$ decay is

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \langle m_\nu \rangle^2$$

The leptonic phase space factors G are accurately calculable.

Nuclear matrix elements for Double Beta Decay are as important as the data to determine Neutrino Mass.

2νββ decay : Description of the Gamow-Teller amplitudes

The double Gamow-Teller transition from g.s.to g.s.:

$$M_{GT}^{2\nu} = \sum_{m_i m_f} \frac{\langle 0_f^+ \| \beta^- \| 1_{m_f}^+ \rangle \langle 1_{m_i}^+ \| \beta^- \| 0_i^+ \rangle}{(E^{m_f} + E^{m_i})/2}$$

0ν decay :

All multipoles contribute(0^+,1^-, ……), enhanced role of nucleon short range correlations.

$$|M_{0\nu}| \equiv M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu} = \left\langle f \left| \sum_{lk} H(r_{lk}, \overline{A}) \tau_l^+ \tau_k^+ \left(\vec{\sigma}_l . \vec{\sigma}_k - \frac{g_V^2}{g_A^2}\right) \right| i \right\rangle$$

⇑ Neutrino Potential

Why to focus on the nuclear deformation?

V.Rodin, A. Faessler, F. Šimkovic., P. Vogel, NPA793 (2007) (Spherical QRPA)

Nuclear transition	$\langle M^{0\nu} \rangle$ RQRPA	$\langle M^{0\nu} \rangle$ QRPA	$T_{1/2}^{0\nu}(\langle m_{\beta\beta} \rangle = 50 meV)$ [yrs]
$^{76}Ge \rightarrow ^{76}Se$	3.92	4.51	8.60×10^{26}
$^{100}Mo \rightarrow ^{100}Ru$	2.78	3.34	2.37×10^{26}
$^{130}Te \rightarrow ^{130}Xe$	2.95	3.66	2.16×10^{26}
$^{150}Nd \rightarrow ^{150}Sm$	4.16	4.74	2.23×10^{25}

Strongly deformed

Shell Model calculations can forget about it !

- $2\nu\beta\beta$ in deformed nuclei; QRPA with schematic separable forces

  F. Šimkovic, L. Pacearescu, A. Faessler, NPA **733** (2004)

  R. Alvarez-Rodriguez *et al.*, PRC **70** (2004)

In $0\nu\beta\beta$ arise a problem of how to fix numerous strength parameters of the forces in different J^{π} partial channels.

- QRPA with realistic forces in deformed nuclei

  to attack $0\nu\beta\beta$ of ^{150}Nd; applied first to $2\nu\beta\beta$

M. Saleh Yousef , V.Rodin, A.Faessler and F. Šimkovic arXiv:0806.0964 [nucl-th]

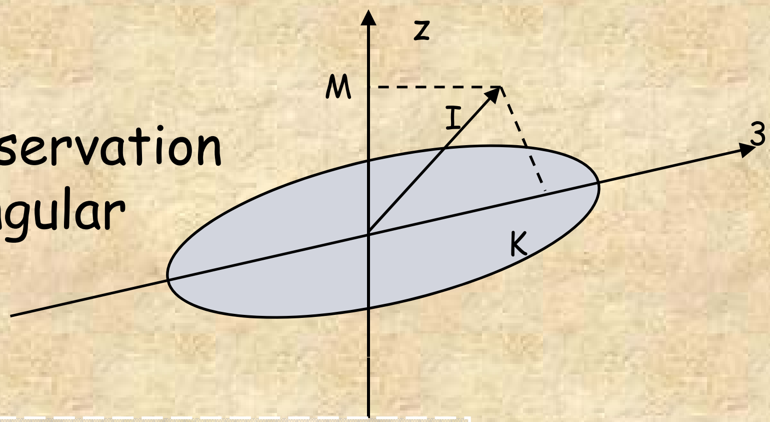
Also

g_{pp} fitted to $2\nu\beta\beta$-decay half-life $\Rightarrow$ stable $M^{0\nu}$

Deformed Nuclei:

Use of deformed single particle basis

⇨ the s.p. wave function without conservation of the total angular momentum, only angular momentum projection (K) is conserved.



$$M_{GT}^{2\nu} = \sum_{m_i m_f} \sum_{K=0,\pm 1} \frac{\langle 0_f^+ \| \beta_K^- \| 1_{m_i}^+(K) \rangle \langle 1_{m_f}^+(K) | 1_{m_i}^+(K) \rangle \langle 1_{m_i}^+(K) \| \beta_K^- \| 0_i^+ \rangle}{(E_K^{m_f} + E_K^{m_i})/2}$$

The overlap is necessary since the two sets of intermediate states calculated within the QRPA are not the same.

Where,

$$\beta_K^- = \sum_{pn} \langle p | \tau^+ \sigma_K | n \rangle a_p^+ a_n$$

GT transition operator

QRPA Formalism (deformed nuclei)

QRPA wave functions of GT excitations for even-even nuclei in the laboratory frame

$$|1M(K),m\rangle = \sqrt{\frac{3}{16\pi^2}}\left[D^1_{MK}(\varphi,\vartheta,\psi)Q^{m^+}_{K} + (-1)^{1+K}D^1_{M-K}(\varphi,\vartheta,\psi)Q^{m^+}_{-K}\right]|RPA\rangle \qquad (K=\pm 1)$$

$$|1M(K),\, m\rangle = \sqrt{\frac{3}{8\pi^2}}\left[D^1_{MK}(\varphi,\vartheta,\psi)Q^{m^+}_{K}\right]RPA\rangle \qquad (K=0)$$

The intrinsic states are generated by the phonon creation operator

$$Q_K^{m\dagger} = \sum_i \left(X_{i,K}^m A_i^\dagger(K) - Y_{i,K}^m \widetilde{A}_i(K) \right)$$

$$A_{pn}^\dagger(K) = \alpha_{p\rho_p}^\dagger \widetilde{\alpha}_{n\rho_n}^\dagger \quad , \quad [A_i, A_j^\dagger] = \delta_{ij} + X$$

The quasi particle creation and annihilation operators can be defined by the Bogolyubov transformation

$$\begin{pmatrix} \alpha_\tau^\dagger \\ \widetilde{\alpha}_\tau \end{pmatrix} = \begin{pmatrix} u_\tau & v_\tau \\ -v_\tau & u_\tau \end{pmatrix} \begin{pmatrix} a_\tau^\dagger \\ \widetilde{a}_\tau \end{pmatrix}$$

Excitation energy and forward-and backward-going amplitudes - by solving QRPA matrix equation:

$$\begin{pmatrix} \mathcal{A}(K) & \mathcal{B}(K) \\ -\mathcal{B}(K) & -\mathcal{A}(K) \end{pmatrix} \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix} = \omega_K^m \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix}$$

where

$$\mathcal{A}_{ij}(K) = \langle RPA | [A_i, [H, A_j^\dagger]] | RPA \rangle$$

$$\mathcal{B}_{ij}(K) = -\langle RPA | [A_i, [H, \widetilde{A}_j]] | RPA \rangle$$

Nuclear Hamiltonian

The total Hamiltonian is defined as: $H = H_o + H_{int}$

$$H_o = \sum_{\tau} \varepsilon_{\tau} a_{\tau}^{\dagger} a_{\tau} \quad (\tau = p, n)$$ Single particle part

$$H_{int} = \sum_{pnp'n'} V_{pn\,p'n'} a_p^{\dagger} a_n^{\dagger} a_{n'} a_{p'}$$ Residual two body interaction

Single Particle States

Single-particle wavefunctions in deformed Woods-Saxon potential :

$$\left|\tau\rho_\tau\right\rangle = \sum_{N_d n_z}[b^{(+)}_{N n_z \Omega_\tau}\left|(N_d n_z \Lambda_\tau), \Omega_\tau = \Lambda_\tau + 1/2\right\rangle\left|\Sigma = 1/2\right\rangle$$

$$+ b^{(-)}_{N n_z \Omega_\tau}\left|(N_d n_z \Lambda_\tau + 1), \Omega_\tau = \Lambda_\tau + 1 - 1/2\right\rangle\left|\Sigma = -1/2\right\rangle]$$

Deformed harmonic oscillator wave function

Spin wave function

The selection rules $\Omega_p - \Omega_n = K$ for K= 0, ±1 and $\pi_p\pi_n = 1$

$$\mathcal{A}_{p_1n_1p_2n_2}(K) = (E_{p_1} + E_{n_1})\delta_{p_1p_2}\delta_{n_1n_2} + g_{pp}[V_{p_2\tilde{n}_2p_1\tilde{n}_1}(u_{p_1}u_{n_1}u_{p_2}u_{n_2}) + V_{\tilde{p}_1n_1\tilde{p}_2n_2}(v_{p_1}v_{n_1}v_{p_2}v_{n_2})]$$

$$- g_{ph}[V_{p_2n_1p_1n_2}(u_{p_2}v_{n_2}v_{n_1}u_{p_1}) + V_{p_1n_2p_2n_1}(v_{p_1}u_{n_1}v_{p_2}u_{n_2})]$$

$$\mathcal{B}_{p_1n_1p_2n_2}(K) = -g_{pp}[V_{p_2\tilde{n}_2p_1\tilde{n}_1}(v_{p_1}v_{n_1}u_{p_2}u_{n_2}) + V_{\tilde{p}_1n_1\tilde{p}_2n_2}(u_{p_1}u_{n_1}v_{p_2}v_{n_2})]$$

$$- g_{ph}[V_{p_2n_1p_1n_2}(v_{p_1}u_{n_1}u_{p_2}v_{n_2}) + V_{p_1n_2p_2n_1}(v_{p_2}u_{n_2}u_{p_1}v_{n_1})]$$

where $E_\tau = \sqrt{\varepsilon_\tau^2 + \Delta_\tau^2}$

Realistic Interaction:

As a realistic two-body interaction we use the nuclear matter G-matrix.

$$G = V + V \frac{Q}{W - H_o + i\varepsilon} G$$

Bethe-Goldstone equation

The G-matrix in the deformed single particle basis:

$$\left|N_d n_z \Lambda_\tau\right\rangle = \sum_{N_o} \sum_{l} A^{N_o l n_r}_{N_d n_z \Lambda_\tau} \left|N_o l \Lambda_\tau\right\rangle$$

Wave function in deformed basis

overlap integral

Wave function in spherical basis

Single particle state

$$\left|\tau\rho_\tau\right\rangle = \sum_{N_o l j} B^{(\tau)}_{N_o l j} \left|(N_o l j), \Omega_\tau\right\rangle$$

deformed Woods-saxon state

spherical harmonic oscillator state

Two body wavefunction

$$\left|p\rho_p \bar{n}\rho_n\right\rangle = \sum_{(N_o l j)_p} \sum_{(N_o l j)_n} B^{(p)}_{(N_o l j)_p} B^{(n)}_{(N_o l j)_n} \sum_{J} C^{JK}_{j_p \Omega_p j_n \Omega_n} \left|(N_o l j)_p (N_o l j)_n, JK\right\rangle$$

The G-matrix in the deformed single particle basis:

$$\left\langle p\rho_p \bar{n}\,\rho_n \right| G \left| p'\rho_{p'} \bar{n}'\rho_{n'} \right\rangle = \sum_{J} \sum_{(N_o l j)_p} \sum_{(N_o l j)_n} \sum_{(N_o l j)_{p'}} \sum_{(N_o l j)_{n'}} B^{(p)}_{(N_o l j)_p} B^{(n)}_{(N_o l j)_n} B^{(p')}_{(N_o l j)_{p'}} B^{(n')}_{(N_o l j)_{n'}}$$

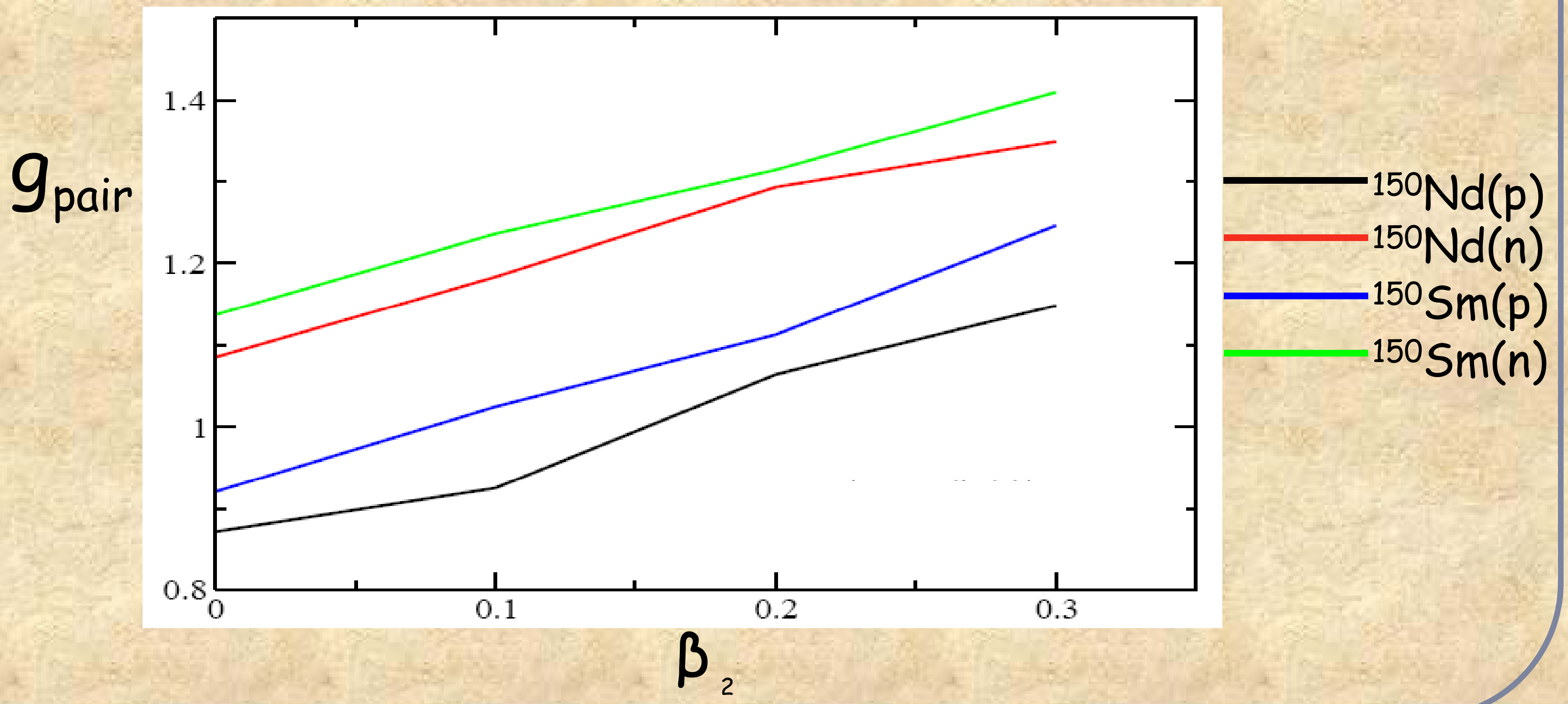
$$\times \quad (-1)^{j_n - \Omega_n} \quad (-1)^{j_n' - \Omega_n'} \; C^{JK}_{j_p \Omega_p j_n \Omega_n} C^{JK}_{j_{p'} \Omega_{p'} j_{n'} \Omega_{n'}}$$

$$\times \quad \left\langle (N_o l j)_p (N_o l j)_n , J \right| G \left| (N_o l j)_{p'} (N_o l j)_{n'} , J \right\rangle$$

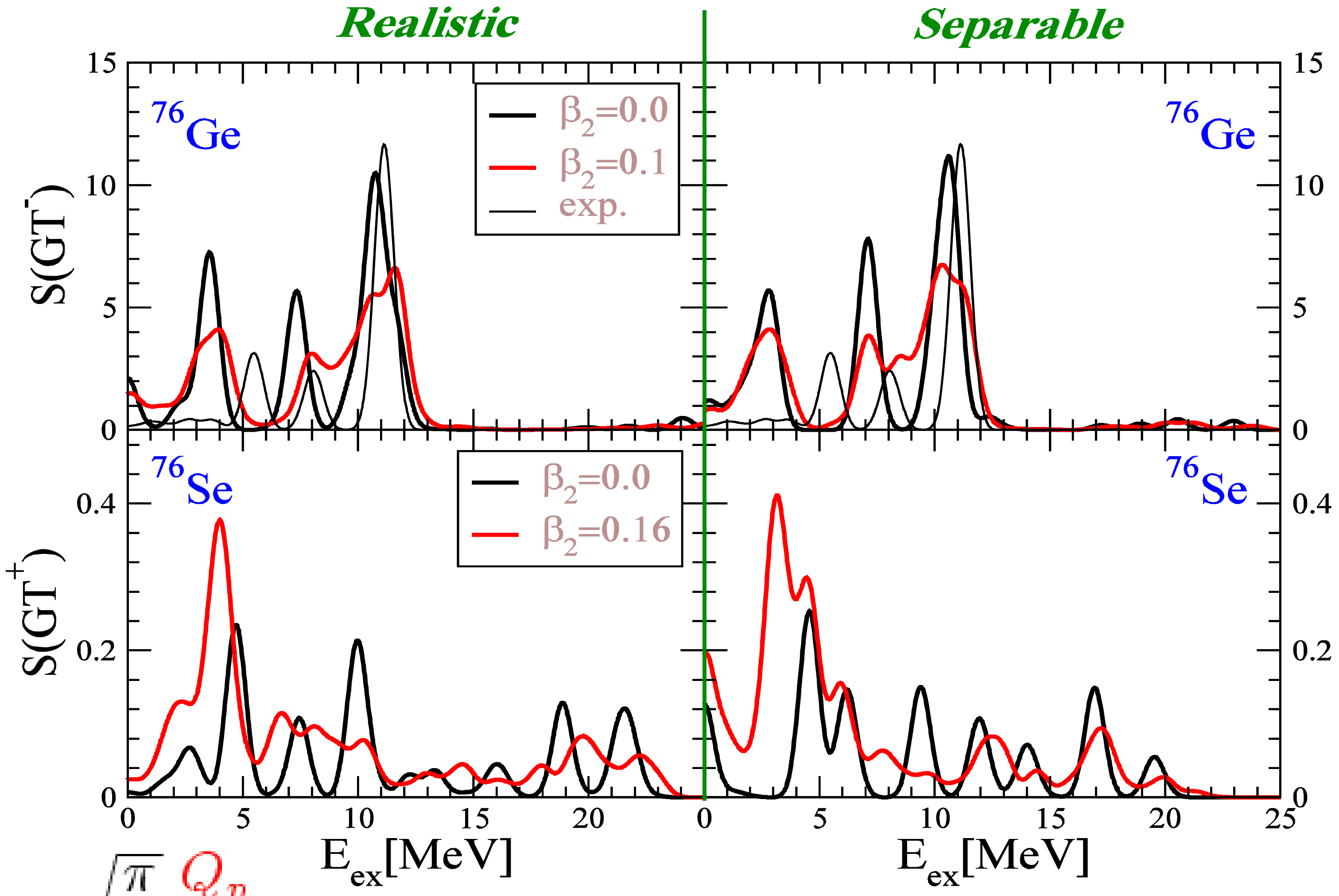
G-matrix elements in
spherical single particle basis
Bonn CD potential

Results

- The BCS equation is solved and the strenghts g^{p}_{pair} and g^{n}_{pair} are determined to reproduce the experimental odd-even mass difference for each nucleus

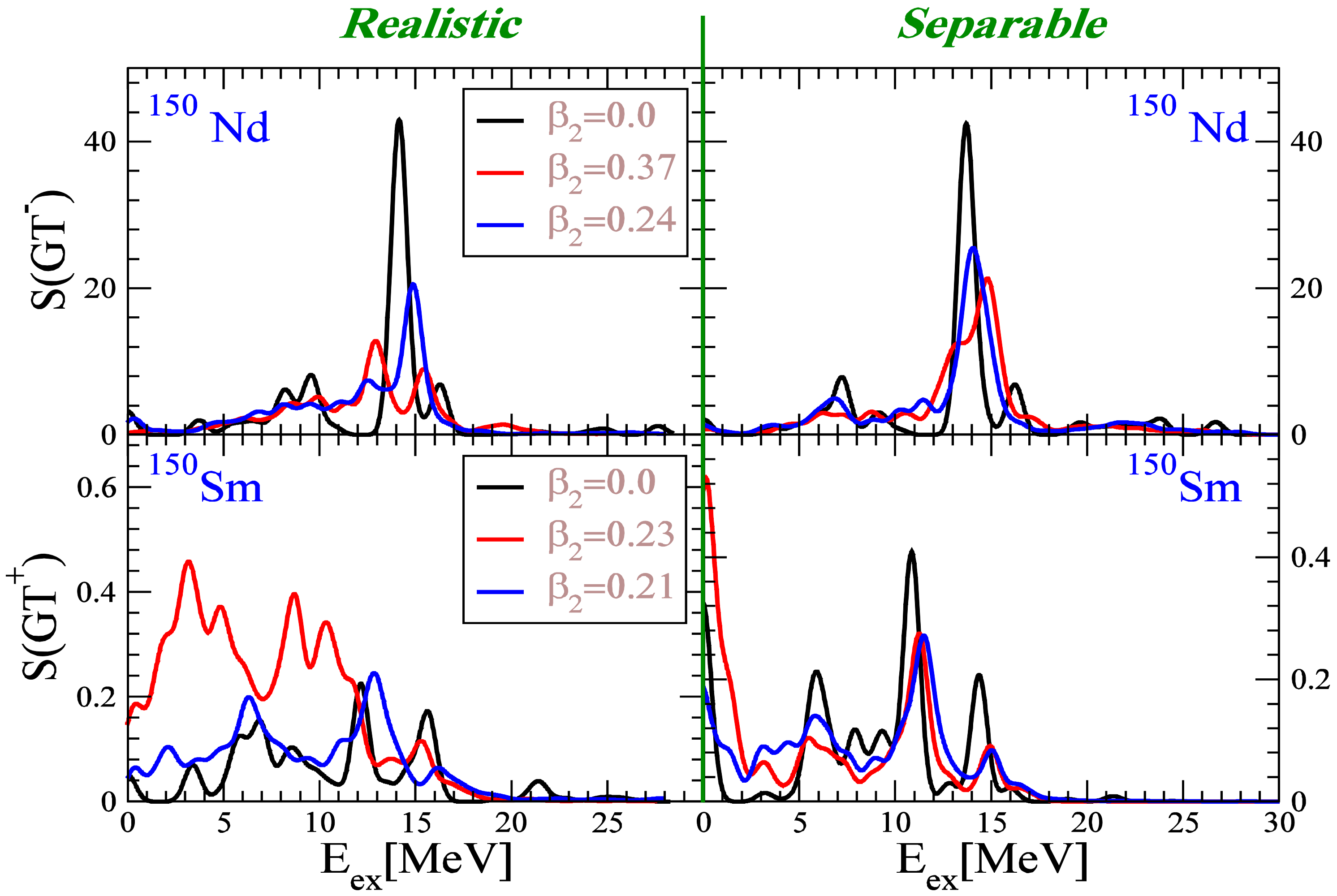


GT strength functions

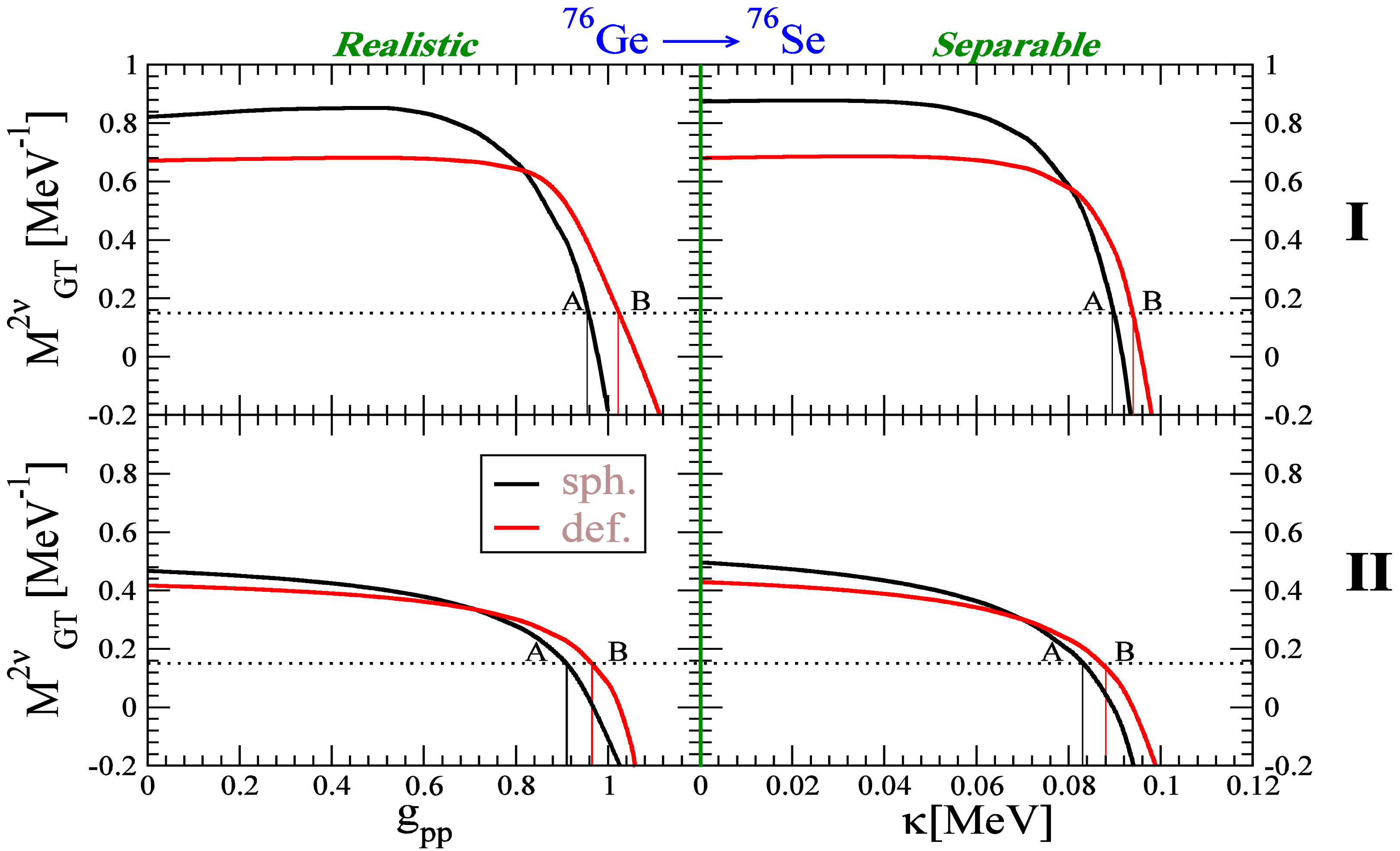


$$\beta_2 = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

GT strength functions

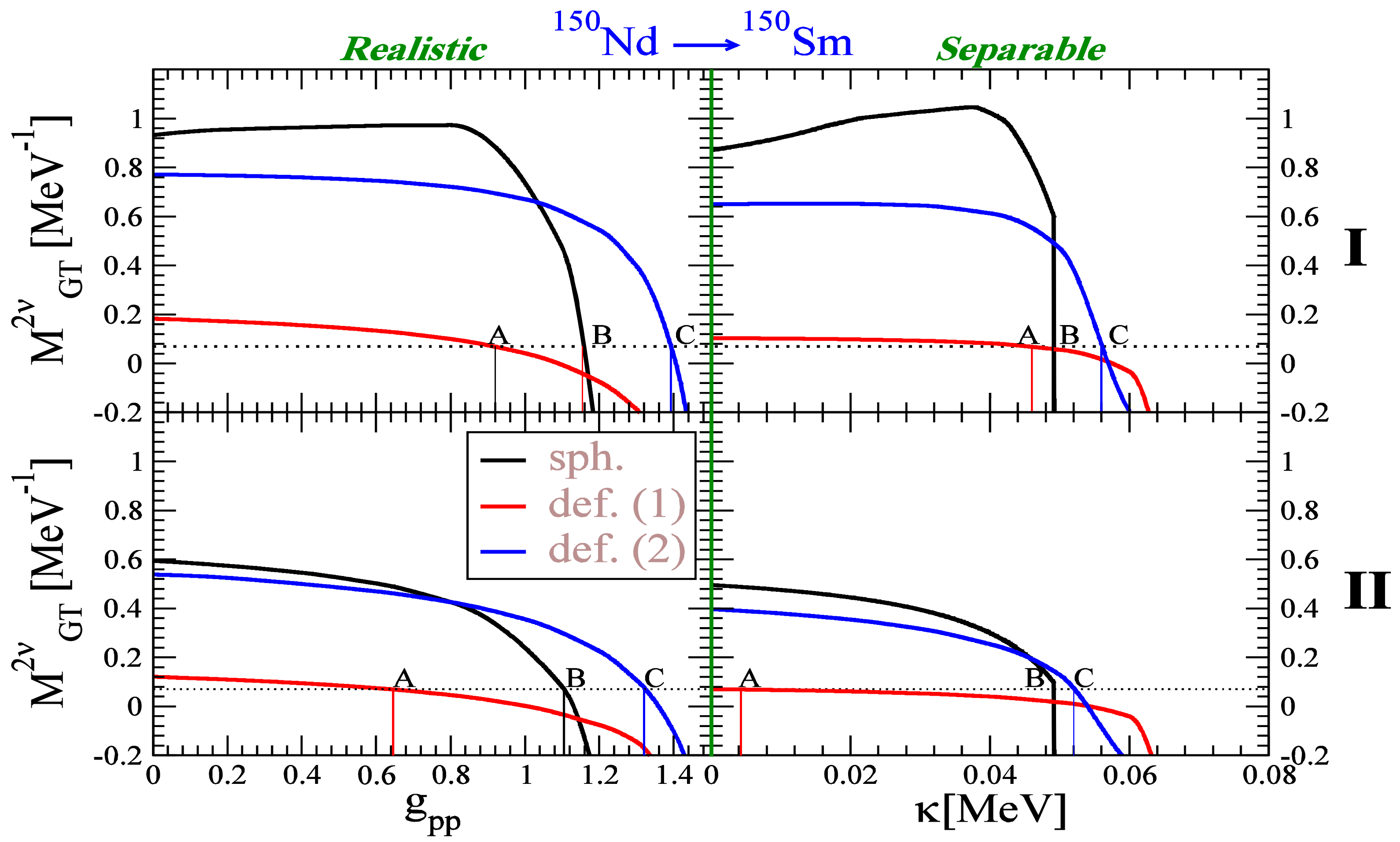


Matrix Elements



def.= exp. defor.: $\beta_2(^{76}Ge)$= 0.1, $\beta_2(^{76}Se)$= 0.16 (P. Raghavan, At. Data Nucl. Data Tabl. **42** (1989))

Matrix Elements



def. I — exp. defor.: $\beta_2(^{150}Nd) = 0.37 \pm 0.09$, $\beta_2(^{150}Sm) = 0.23 \pm$
(P. Raghavan, At. Data Nucl. Data Tabl. **42** (1989))

def. II — calc. defor.: $\beta_2(^{150}Nd) = 0.24$, $\beta_2(^{150}Sm) = 0.21$
(P. Moeller et al., At. Data Nucl. Data Tabl. **59** (1995))

Conclusions

- BCS equations are solved for protons and neutrons. The protons and neutron gaps are determined to reproduce the odd-even mass differences.
- G-matrix in deformed s.p. basis is calculated using that for spherical s.p. basis.
- The QRPA equation is solved using the realistic forces and B(GT) strengths are calculated for ^{76}Ge ,^{76}Se, ^{150}Nd and ^{150}Sm nuclei considering deformation.
- $2\nu\beta\beta$ matrix element for ^{76}Ge and ^{150}Nd with deformation have been obtained

Outlook

- Go Further to study the matrix elements of $0\nu\beta\beta$ decay within deformed QRPA and realistic forces.

THANK YOU