The effect of structure formation on the expansion of the universe

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A factor of 2 in distance

- The early universe is well described by a model which is homogeneous and isotropic, contains only ordinary matter and evolves according to general relativity.
- However, such a model underpredicts the distances measured in the late universe by a factor of 2.
- This is interpreted as faster expansion.
- There are three possibilities:

1) There is matter with negative pressure.

2) General relativity does not hold.

3) The universe is not homogeneous and isotropic.

Possibilities

- By introducing exotic matter or modified gravity it is possible to explain the distance observations.
- Such models suffer from **the coincidence problem.** Why has the acceleration started recently, i.e. why $\rho_{de} \sim \rho_m$ today?
- More importantly, linearly perturbed FRW models do not include non-linear structures.
- Before concluding that new physics is needed, we should take into account the known breakdown of homogeneity and isotropy.

Backreaction

- **The average evolution of an inhomogeneous and/or anisotropic spacetime is not the same as the evolution of the corresponding smooth spacetime.**
- At late times, non-linear structures form, and the universe is only statistically homogeneous and isotropic, on scales above 100 Mpc.
- Finding the model that describes the average evolution of the clumpy universe was termed **the fitting problem** by George Ellis in 1983.

Backreaction, exactly

- Consider a dust universe. The Einstein equation is $G_{\alpha\beta} = 8\pi G_N \rho u_\alpha u_\beta$.
- The (exact, local, covariant) scalar part is:

$$\begin{cases} \dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2 + 2\omega^2 \\ \frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2 - \omega^2 \end{cases}$$

$$\dot{\rho} + \theta\rho = 0$$

- Here θ is the expansion rate, ρ is the energy density, $\sigma^2 \geq 0$ is the shear, $\omega^2 \geq 0$ is the vorticity and ${}^{(3)}R$ is the spatial curvature.
- We take $\omega^2 = 0$.

- The Buchert equations (1999):

$$\begin{cases} 3\frac{\ddot{a}}{a} = -4\pi G\langle\rho\rangle + Q \\ 3\frac{\dot{a}^2}{a^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle{}^{(3)}R\rangle - \frac{1}{2}Q \\ \partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0 \end{cases}$$

$$\dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2$$
$$\frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2$$
$$\dot{\rho} + \theta\rho = 0$$

- Here $a(t) \propto \left(\int d^3x\sqrt{{}^{(3)}g}\right)^{1/3}$. The backreaction variable is

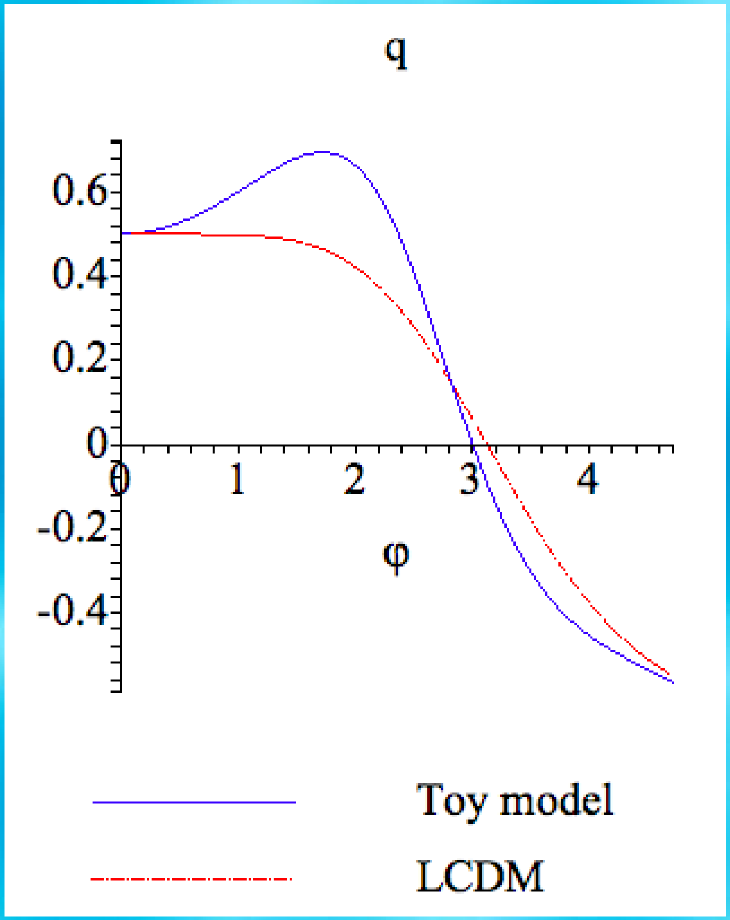
$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle$$

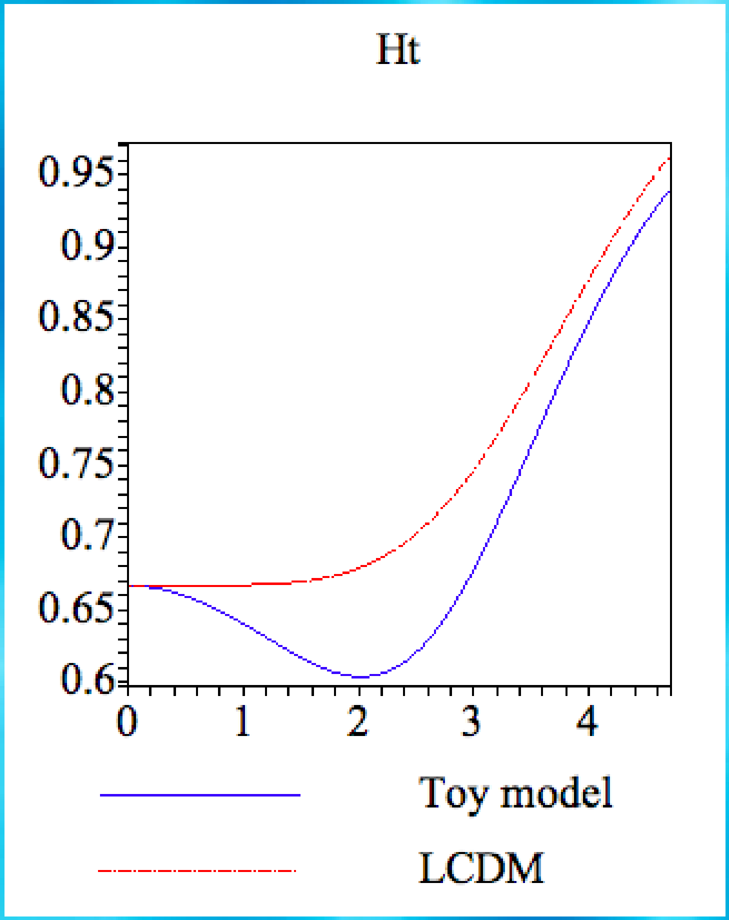
$$\langle f\rangle \equiv \frac{\int d^3x\sqrt{{}^{(3)}g}\,f}{\int d^3x\sqrt{{}^{(3)}g}}$$

- The average expansion can accelerate, even though the local expansion decelerates.

Demonstrating acceleration

- The average expansion rate can increase, because the fraction of volume occupied by faster expanding regions grows.
- Structure formation involves overdense regions slowing down and underdense regions speeding up.
- Consider a toy model with one overdense and one underdense region, both described by the spherical collapse model.
- For an empty void we have $a_1 \propto t$, for an overdense region we have $a_2 \propto 1\text{-}\cos\varphi$, $t \propto \varphi\text{-}\sin\varphi$.
- The overall scale factor is $a = (a_1^3 + a_2^3)^{1/3}$.





$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_1 + \frac{a_2^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_2 = v_1 H_1 + v_2 H_2$$

$$q \equiv -\frac{1}{H^2} \frac{\ddot{a}}{a} = \frac{H_1^{\ 2}}{H^2} v_1 q_1 + \frac{H_2^{\ 2}}{H^2} v_2 q_2 - 2 v_1 v_2 \frac{(H_1 - H_2)^2}{H^2}$$

Towards reality

- Acceleration due to structures is possible: is it realised in the universe?
- The non-linear evolution is too complex to follow exactly.
- Because the universe is statistically homogeneous and isotropic, a statistical treatment is sufficient.
- We can evaluate the expansion rate with an evolving ensemble of regions.

QuickTime™ and a
decompressor
are needed to see this picture.

The peak model

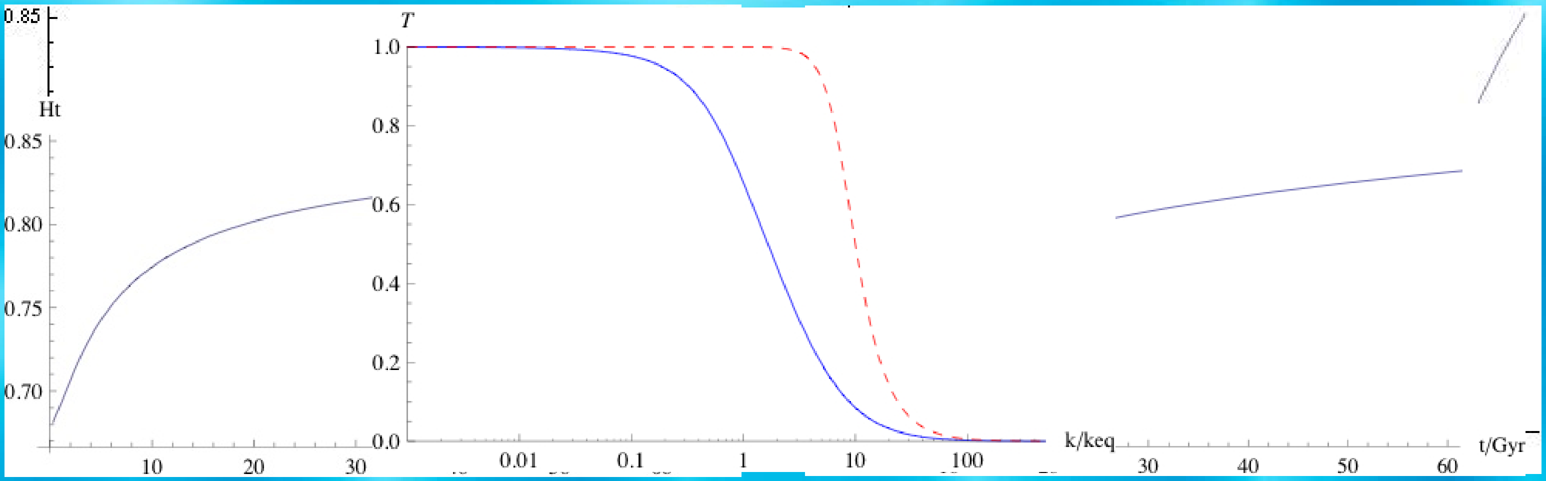
- We start from a FRW background of dust with an initial Gaussian linear density field.
- We identify structures with spherical isolated peaks of the smoothed density field. (BBKS 1986)
- We keep the smoothing threshold fixed at $\sigma(t,R)=1$, which gives the time evolution $R(t)$.
- Each peak expands like a separate FRW universe.
- The peak number density as a function of time is determined by the primordial power spectrum and the transfer function.
- We take a scale-invariant spectrum with cold dark matter transfer function.

- The expansion rate is $H(t) = \int_{-\infty}^{\infty} d\delta \, v_\delta(t) H_\delta(t)$.

- There are no parameters to adjust.

- Consider two approximate transfer functions.

  Bonvin and Durrer BBKS



Ht as a function of time (in Gyr, with t_{eq}=100 000 yr)

Ht as a function of R/R_{eq}

Conclusion

- Observations of the late universe are inconsistent with a homogeneous and isotropic model with ordinary matter and gravity.
- FRW models do not include non-linear structures.
- The Buchert equations show that the average expansion of a clumpy space can accelerate.
- Modelling evolving structures with the peak model, the expansion rate Ht rises by 10-30% around $10^5\, t_{eq}$.
- Many things are missing: trough-in-a-peak problem, correct transfer function, non-spherical evolution, ...
- The average geometry has to be related to light propagation.