Aspects of Coulomb Gauge QCD at Zero and Finite Temperature

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Graduate Workshop 08, Blaubeuren

A Comment on Gauge-Fixing

General Remarks on gauge-fixing:

- Gauge-fixing picks certain features of theory – chooses subset of the space $\mathcal{A}$ of all gauge configurations.
- Same physical effect may look very different in different gauges. Description of an effect may be especially easy in a certain gauge.
- Gauge-dependent quantities may look quite “unphysical” (scalar fermions, instantaneous potentials, . . .)

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For certain tasks other gauges may be more useful.

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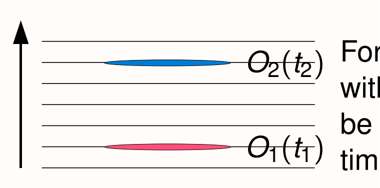

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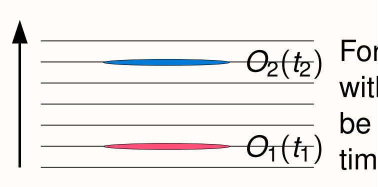

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All gauge-variant quantities washed away by averaging over all configurations, only gauge-invariant ("physical") quantities propagate $\overset{\text{usual arg.}}{\longrightarrow}$ positive state space.

Basic Facts about Coulomb gauge (II)

- Positive definite state space allows variational calculations

- Gauß' law

$$(\delta^{ab}\partial_i + gf^{acb}A_i^c)E_i^b = -g\rho_q^a$$

directly implemented, related to vanishing of total color charge, [Reinhardt, Watson; 2008]

- Gauge-fixing at different time-slices independent $\rightarrow$ allows independent rotations in color space $\langle A^a(\mathbf{x}, t_1)B^b(\mathbf{y}, t_2)\rangle = R^{aa'}(t_1)R^{bb'}(t_2)\langle A^{a'}(\mathbf{x}, t_1)B^{b'}(\mathbf{y}, t_2)\rangle$ correlators of (fundamentally) color-charged fields vanish for unequal times, [Baulieu, Zwanziger; 1999]

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- Even perturbative renormalizability not (yet) proven
- Non-covariance makes calculations cumbersome.
- Statement about Confinement – Zwanziger inequality, [Zwanziger; 2003]:
  Write A_0-A_0propagator as

  $$D_{00}(x) = V_C(\boldsymbol{x})\,\delta(t) + (\text{non.-inst.})$$

  One can show that for large $|\boldsymbol{x}|$

  $$V_{\text{Wilson}}(\boldsymbol{x}) \leq -\frac{4}{3} V_C(\boldsymbol{x})$$

  No Confinement without Coulomb Confinement
- V_{Wilson} lin. rising $\rightarrow$ $V_C(\boldsymbol{x})$ (at least) lin. rising
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The Gribov-Zwanziger Scenario (I)

- Gauge-fixing not unique in non-Abelian gauge theory – Gribov problem
- Initial proposal, [Gribov; 1977]: Restrict functional integral to Gribov region Ω, $\mathcal{M}_{FP} = -\partial \cdot D > 0$
- Later refinement, [Semenov-Tyan-Shanskii, Franke; 1982], [Zwanziger; 1982]: Gribov copies even inside Ω, restrict functional integral to Fundamental Modular Region Λ (nontrivial topology, [van Baal; 1992])

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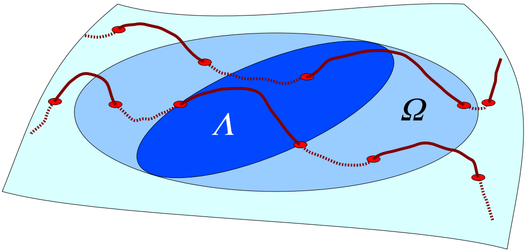

The Gribov-Zwanziger Scenario (II)

- Ω is infinite-dimensional convex region $\longrightarrow$ boundary dominates
- $\lambda_{\min}(\mathcal{M}_{\mathrm{FP}}) \longrightarrow 0$, enhanced ghost and suppressed (transverse) gluon propagation for small momenta – Gribov-Zwanziger confinement scenario, [Gribov; 1977], [Zwanziger; 1989]
- Perturbation theory lives around $A = 0$, knows nothing about the Gribov horizon.
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- Scenario first derived in Coulomb gauge, but not limited to it.

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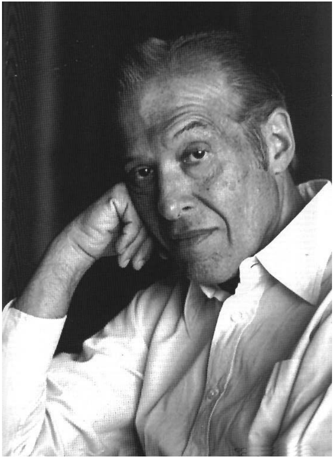
Some Remarks on DSEs (I)

Dyson-Schwinger equations (DSEs)



Some Remarks on DSEs (II)

Basic Idea:

- Lagrangian $\mathcal{L}$ of a system in classical mechanics
- Use variational method to derive Euler-Lagrange equations (equations of motion)
- Solve these differential equations $\longrightarrow$ trajectories
- Is this possible also for Quantum Field Theory?
- No trajectories; solution of theory give in terms of Greens functions
- If we know the Greens functions, we know everything.
- The best we can expect are equations of motion which connect (and so perhaps determine) certain Greens functions $\longrightarrow$ Dyson-Schwinger Equations

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Intuitive Derivation of the Quark DSE (I)

DSEs derived from the action; “handwaving” way:

The quark

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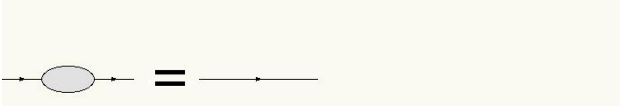

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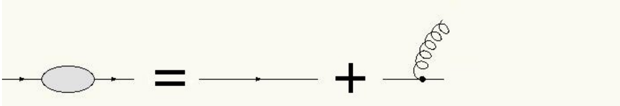

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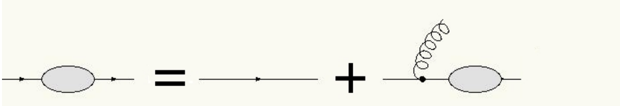

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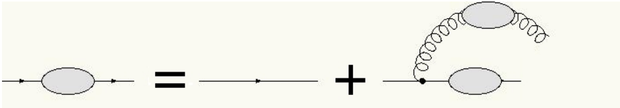

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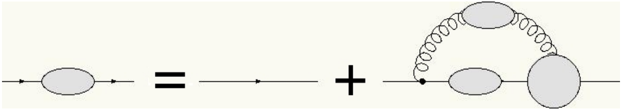

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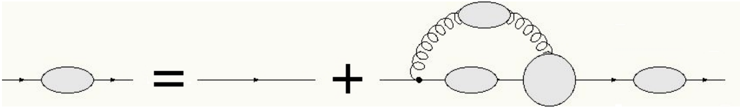

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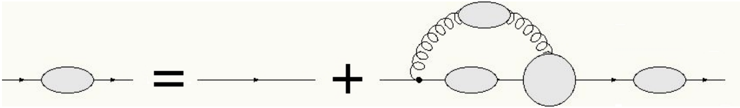

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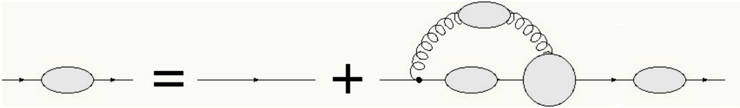

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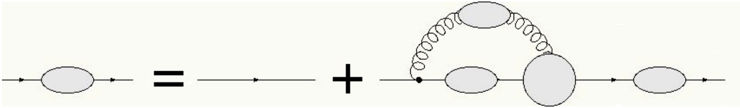

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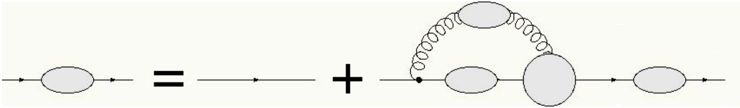

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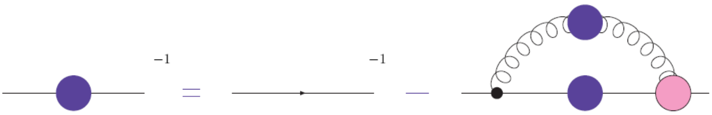

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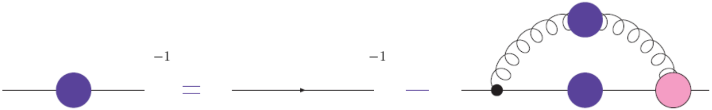

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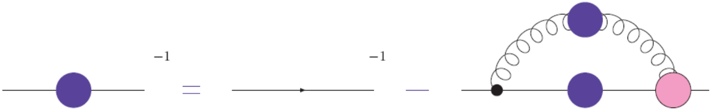

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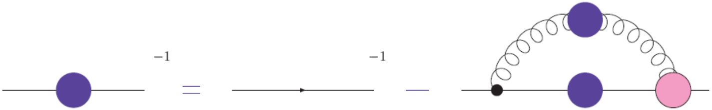

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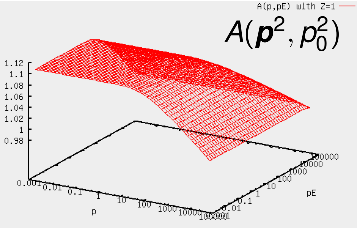

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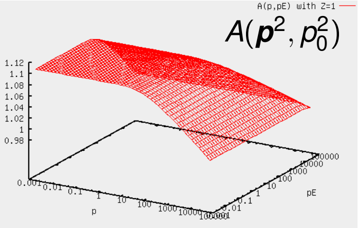

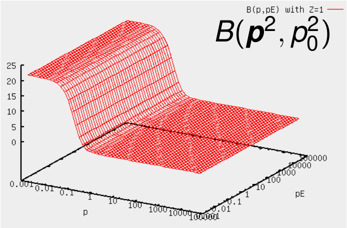

A and D vary with p_0, but suppressed; B and C are constant w.r.t. $p_0 \overset{\mathrm{FT}}{\to} \delta(t)$ – localization in time

Summary and Discussion

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- $\mu_{IR} \to 0$: Colored fields get localized on single time-slices and do not propagate in time. Gauge-variant quantities vanish or diverge.
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K.L., D. Zwanziger, Phys. Rev. D 78, 034038 (2008)

TASK: Determine properties of high-T QCD

- straightforward approach – perturbation theory
- hope: high temperature T

  $\rightarrow$ large momentum transfer $\mu \sim 2\pi T$

  $\rightarrow$ small coupling constant $g(\mu)$

  $\rightarrow$ reliable perturbative expansion

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- Nonanalyticities yield terms $\mathcal{O}(g^3)$, $\mathcal{O}(g^4 \ln g)$, ...
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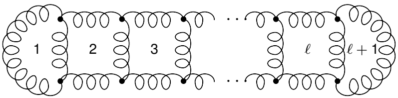

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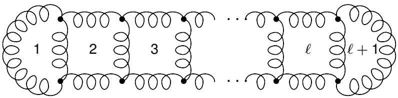

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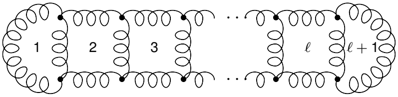

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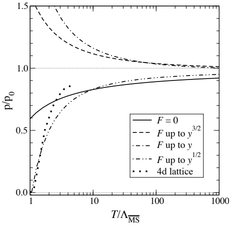

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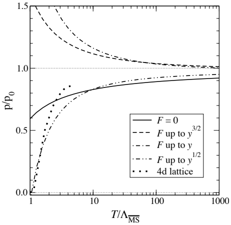

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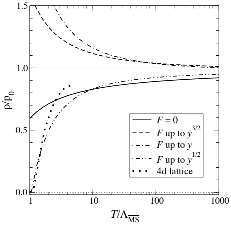

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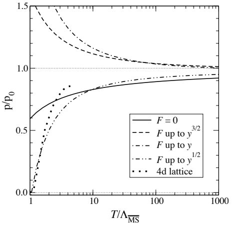

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Pressure from the Gribov Horizon (I)

- Idea: explicitely include contributions from Gribov horizon in a (semi)perturbative approach also for finite T, [Zwanziger; 2006]
- access to confined sector even in “deconfined” phase
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with $u(x)=\sqrt{x^2+\frac{1}{x^2}}$ and $\mu(T)\sim 2\pi T$

- Asymptotic expansion: $m \sim g^2 T$, $w \sim g^6 T^3$, ...
- Compare expansion with lattice results

$$\frac{w^{(6)}_{\text{np, analyt}}}{g^6 T^3}=\frac{(N^2-1)\,N^3}{10\,368\,\pi^4},\qquad \frac{w^{(6)}_{\text{np, lattice}}}{g^6 T^3}=\frac{(N^2-1)\,N^3}{1\,280\,\pi^4}(1\pm 4)$$

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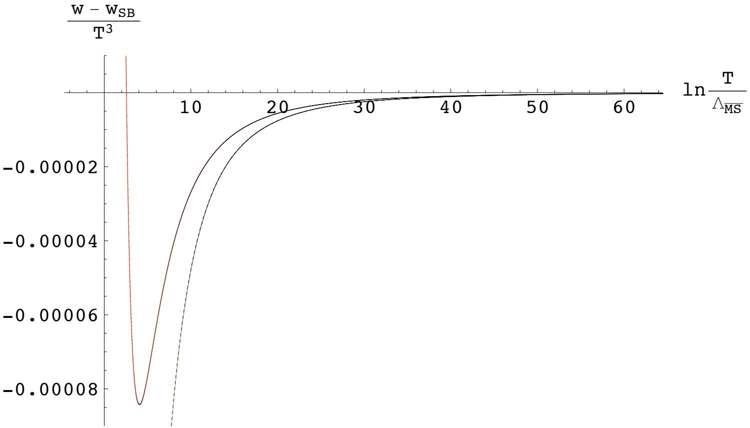
Numerical Results (I)

rescaled free energy $w_r = \frac{w}{T^3}$

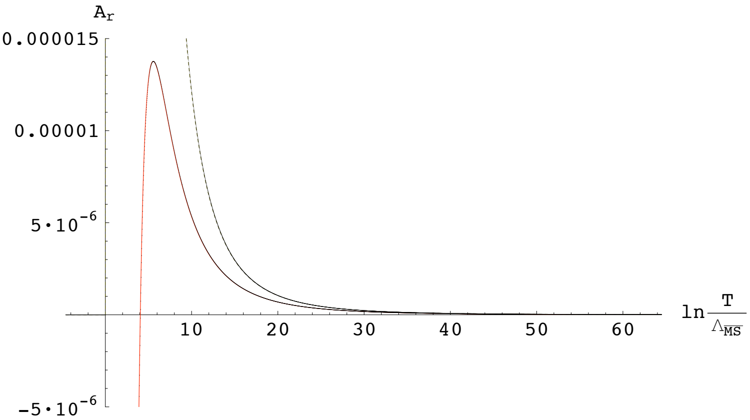
(numerical solution and asymptotic form $\sim g^6$)



Numerical Results (II)

rescaled anomaly $A_r = \frac{e-3p}{T^4} = T\,\frac{\partial}{\partial T}\,w_r$

(numerical solution and asymptotic form $\sim g^8$)



Numerical Results (III)

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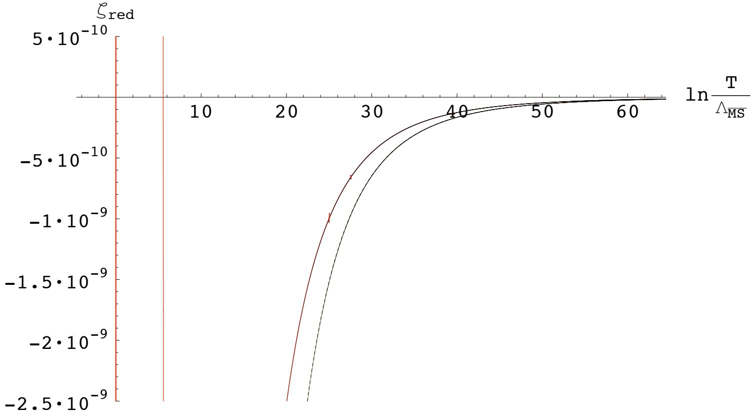

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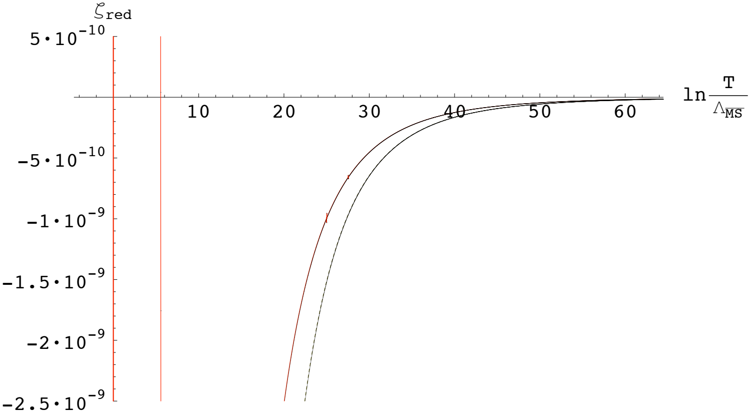

Large deviations: numerical solution ↔ leading order

Summary and Discussion

- Access to nonperturbative sector by cutoff at Gribov horizon – almost purely analytical method
- Manifestation of confinement in "deconfined" phase
- Thermodynamic observables calculable (perturbative results have to be added)
- Results compatible with lattice calculations
- Full results deviate significantly from leading order expression…

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- Additional evidence that series expansions are extremely problematic in hot QCD.

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