Dynamical Processes in the Crust of Neutron Stars: Calculation of Neutrino Mean Free Path

Pavlo Grygorov

with
Prof. Dr. H. Müther
Dr. P. Gögelein
Dr. E.N.E. van Dalen

Institut für Theoretische Physik
Eberhard–Karls–Universität Tübingen

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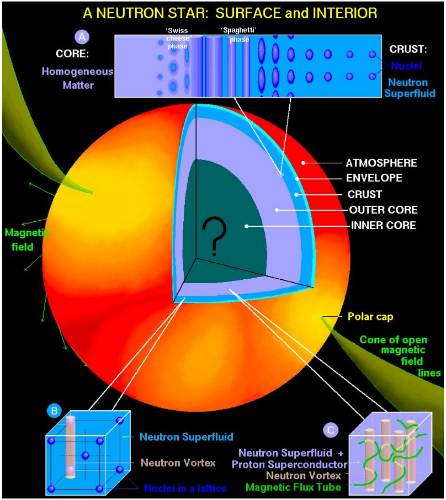
Introduction



Picture by D. Page

- *Inner core:*
  strange baryons?
  quark matter?
- *Outer core:*
  homogenous superfluid neutron and proton matter,
  magnetic flux tube,
- *Crust:*
  nuclei, spaghetti, lasagne, superfluid neutrons,
- *Athmosphere:*
  nuclei, magnetic field.

Skyrme Hartree–Fock Calculations

- SLy4 was used   Chabanat et al., NP A635, 231 (1998)
- The calculations were performed within the cubic Wigner–Seitz cell. It allows to reproduce nonspherical pasta objects: rods, slabs and contains the limit of homogeneous matter in natural way.
- HF equations were solved by using the imaginary time step method

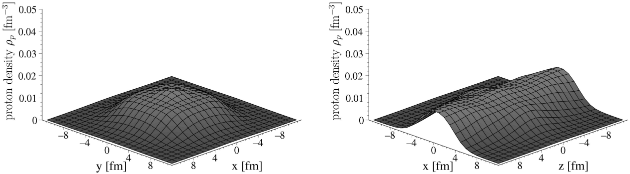
$$\left\{ -\nabla \frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla + U_q(\mathbf{r}) - i\,\mathbf{W}_q(\mathbf{r}) \cdot (\nabla \times \boldsymbol{\sigma}) \right\} \varphi_k^q(\mathbf{r}) = \varepsilon_k^q \, \varphi_k^q(\mathbf{r}, s)\,,$$

- The pairing of nucleons was included by means of density dependent monopole pairing force Garrido et al. PR C60, 064312 (1999)
- Finite temperature effects are considered within FT–HFB Theory.

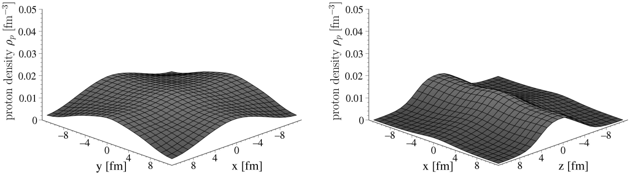
More details   P. Gögelein, PhD Thesis, Tübingen (2007)
P. Gögelein and H. Müther, PR C76, 024312 (2007)

Skyrme Hartree–Fock Calculations

Rod structure



Slab structure



Skyrme Hartree–Fock Calculations

Nonrelativistic Interaction Neutrino with Quasi-Nuclei

Direct URCA processes
(semileptonic reactions)

$$\nu_1 + n_2 \to e_3^- + p_4, \qquad \nu_1 + n_2 \to \nu_3 + n_4$$

The general matrix element has the form

$$M = \frac{G_F C}{\sqrt{2}} J_\mu j^\mu,$$

where

$$J_\mu = i\bar{u}_4(V\gamma_\mu + A\gamma_\mu\gamma_5)u_2, \qquad j^\mu = -i\bar{u}_3\gamma^\mu(1-\gamma_5)u_1$$

Walecka "Th. Nuclear & Subnuclear Physics", (1995)

Skyrme Hartree–Fock Calculations

By using the Fermi's Golden Rule

$$\sigma = \sum_f p_3 E_3 \frac{1}{2} \int_1^{-1} d(\cos\theta) \overline{|M|}^{\,2},$$

$$\overline{|M|}^{\,2} = \frac{G_F^2 C^2}{\pi} \left[V^2 (1 + \cos\theta) |M_1|^2 + A^2 (1 - \frac{1}{3} \cos\theta) |M_2|^2 \right],$$

$$M_1 = \langle \varphi_4 | e^{i\vec{q}\vec{r}} | \varphi_2 \rangle, \quad M_2 = \langle \varphi_4 | \vec{\sigma} e^{i\vec{q}\vec{r}} | \varphi_2 \rangle,$$

where we neglected the lower components in Dirac spinors $u \simeq \binom{\varphi}{0}$

Skyrme Hartree–Fock Calculations

Important Remarks

- The existence of electron sea (blocking factor)

$$\mu_p + \mu_e = \mu_n$$

- V_{cell} is the "unit" volume

$$\frac{1}{\lambda} = \frac{\sigma}{V_{cell}}$$

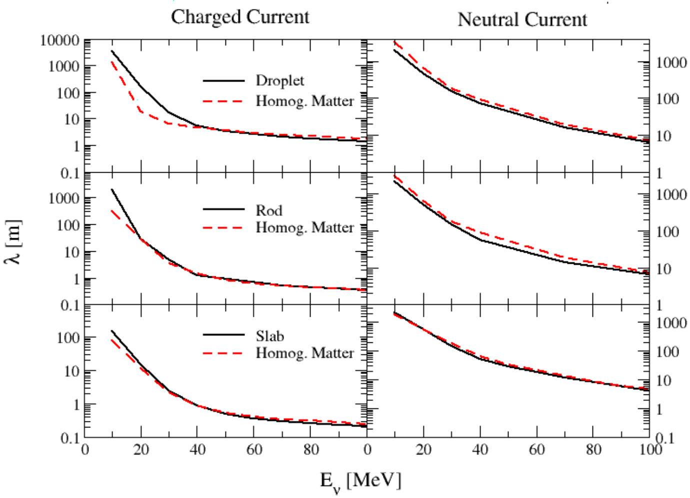
- Absence of spherical symmetry

$$\vec{q}\vec{r} = q_x x + q_y y + q_z z.$$

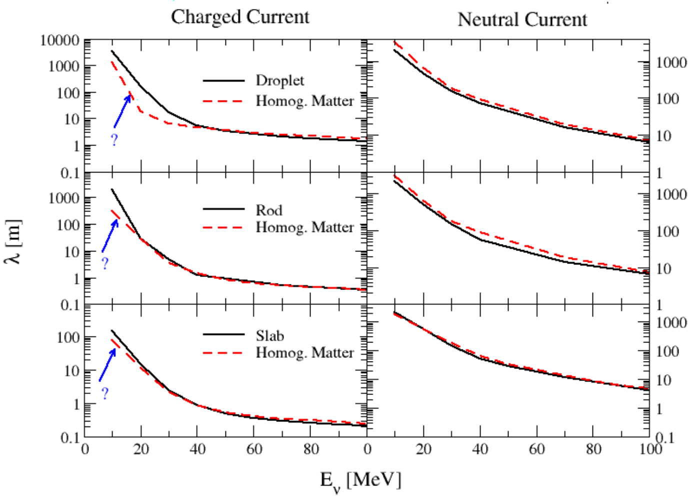
- We approximate

$$\sigma = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

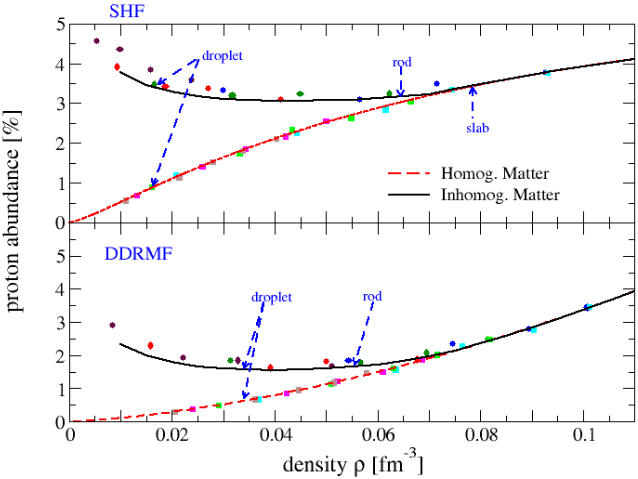
Skyrme Hartree–Fock Calculations (T=1 MeV)



Skyrme Hartree–Fock Calculations (T=1 MeV)



Skyrme Hartree–Fock Calculations



Density Dependent Relativistic Mean–Field Calculations

- Lagrangian density involves σ, ω, δ, ρ mesons by means of density dependent coupling functions: $g_\sigma(\rho)$, $g_\omega(\rho)$, $g_\delta(\rho)$, $g_\rho(\rho)$
- Adjustment of density dependent coupling functions to reproduce self–energy from Dirac Brueckner Hartree–Fock calculations for nuclear matter and finite nuclei.
  E.N.E. van Dalen et al., EPJ A31, 29 (2007)
- Density Dependent Relativistic Mean Field (DDRMF) for nuclear matter, finite nuclei, and the *Wigner–Seitz Cell*.
  P. Gögelein et al., PR C77, 025802 (2008)
- The pairing correlations and finite temperature effects are included as in nonrelativistic SkHF calculations

Density Dependent Relativistic Mean–Field Calculations

Relativistic Interaction Neutrino with Quasi-Nuclei

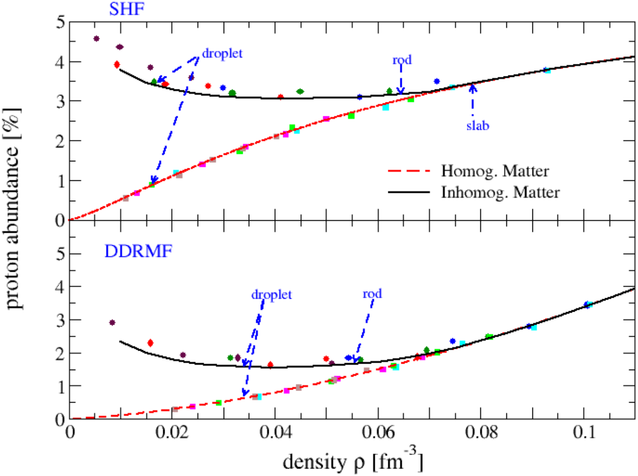
$$M = \frac{G_F C}{\sqrt{2}} J_\mu j^\mu,$$

$$J_\mu^{CC} = i\bar{u}_4[F_1^v(q^2)\gamma_\mu + F_2^v(q^2)\sigma_{\mu\nu}q_\nu + F_A(q^2)\gamma_5\gamma_\mu - iF_p(q^2)\gamma_5 q_\mu]u_2,$$

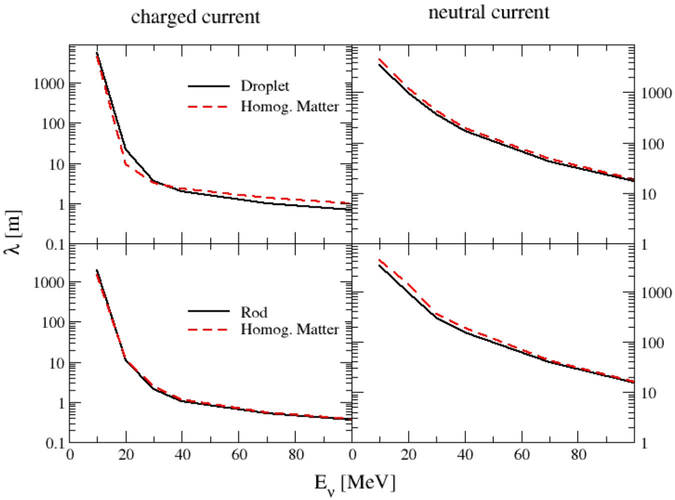
$$\begin{aligned} J_\mu^{NC} = & \frac{i}{2}\bar{u}_4[F_A(q^2)\gamma_5\gamma_\mu - iF_p(q^2)\gamma_5 q_\mu \\ & + (1 - 2\sin^2\theta_W)(F_1^v(q^2)\gamma_\mu + F_2^v(q^2)\sigma_{\mu\nu}q_\nu) \\ & - 2sin^2\theta_W(F_1^s(q^2)\gamma_\mu + F_2^s(q^2)\sigma_{\mu\nu}q_\nu)]u_2, \end{aligned}$$

$$j^\mu = -i\bar{u}_1\gamma^\mu(1-\gamma_5)u_3$$

Density Dependent Relativistic Mean–Field Calculations



Density Dependent Relativistic Mean–Field Calculations (T=1MeV)



Density Dependent Relativistic Mean–Field Calculations

At which Temperature does the "Pasta Phase" disappear?

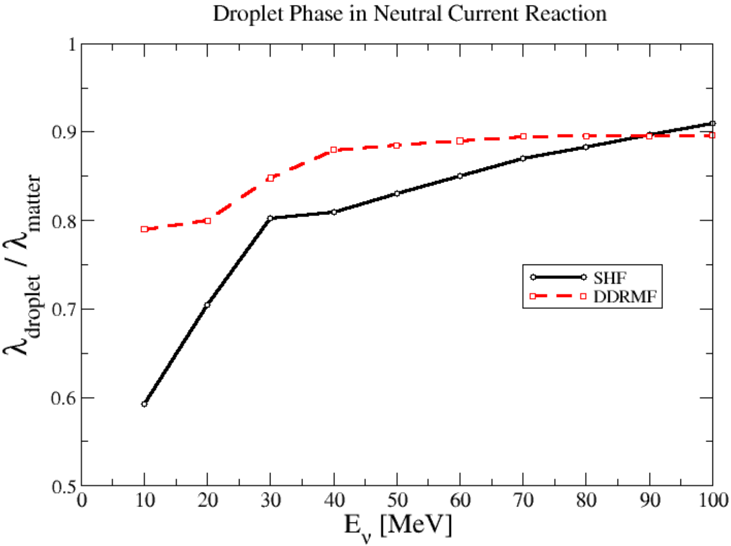
	T_c (MeV)	
	SHF	DDRMF
droplet	$\gtrsim 15$	10
rod	10	5
slab	5	–

- $T_c \gtrsim 15$ MeV from Shen et al., NP A637, 435 (1998)
  Relativistic TF calculations with nonlinear $\sigma-$ and $\omega-$ terms.

Summary

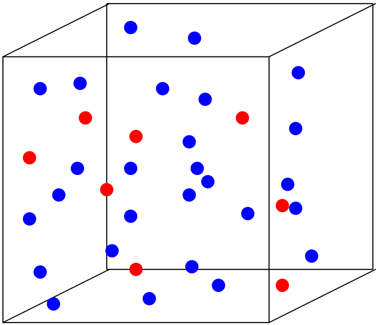
- The relativistic and nonrelativistic interaction of neutrino with "Pasta Phase"
  was investigated both for charged and neutral current reactions.
- It was shown that NMFP is sensitive to the shell effects, which lead to the enhancement of proton abundance in inhomogeneous matter.
- The critical temperature at which the Pasta Phase disappears was found.
- The calculations of the response functions are in progress.

Density Dependent Relativistic Mean–Field Calculations



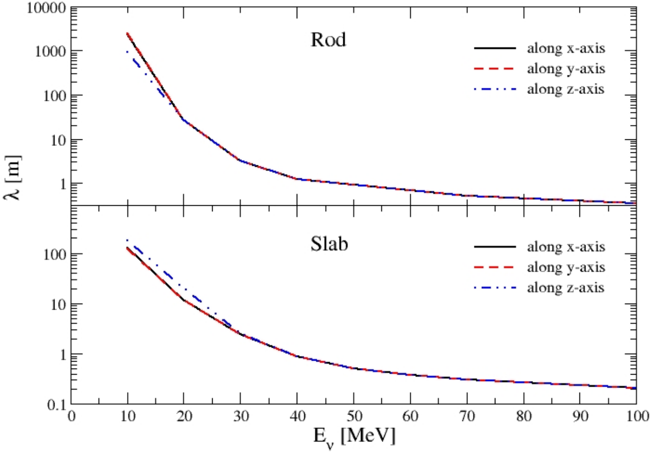
Introduction

Wigner–Seitz Cell



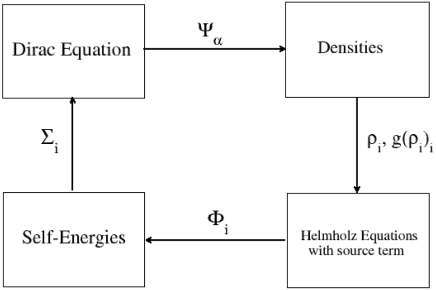
- Neutrons and protons in a Wigner–Seitz Cell as representation of nuclear matter in the crust of neutron stars.
- A cartesian cell is considered so that the whole space can be covered by repeated cells.
- In addition non-spherical structures can be observed.

Angular Dependence of NMFP

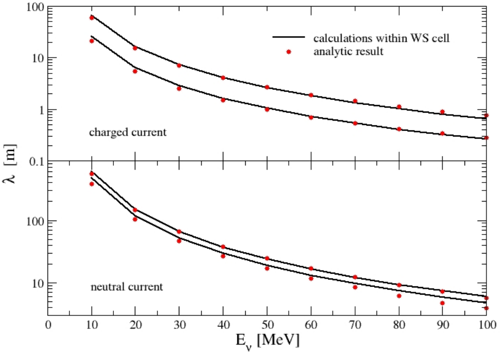


Iterative Procedure

- The Helmholtz equations for the mesons are solved by the *conjugate gradient method*.
- The Dirac equation with the scalar, vector and tensor Hartree self energy is solved by variation applying the *imaginary time step* Gögelein et al., PR C77, 025802 (2008)



Skyrme Hartree–Fock Calculations



$$\frac{\sigma(E_1)}{V} = \frac{1}{\lambda} = \frac{G_F C^2}{4\pi^2}(V^2 + 3A^2) \int_{-\infty}^{E_1} dq_0 \frac{E_3}{E_1}(1 - f_3(E_3)) \int_{|q_0|}^{2E_1 - q_0} dq q S(q, q_0),$$

Density Dependent Relativistic Mean–Field Calculations

Lagrangian dens. consists of three parts: baryon, meson and interaction:

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{int},$$

$$\begin{aligned}
\mathcal{L}_B =& \bar{\Psi}(\, i\gamma_\mu \partial^\mu - M)\Psi, \\
\mathcal{L}_M =& \frac{1}{2}\sum_{i=\sigma,\delta}\left(\partial_\mu \Phi_i \partial^\mu \Phi_i - m_i^2 \Phi_i^2\right) \\
& - \frac{1}{2}\sum_{\kappa=\omega,\rho,\gamma}\left(\frac{1}{2}F_{\mu\nu(\kappa)}F^{\mu\nu}_{(\kappa)} - m_\kappa^2 A_\mu^{(\kappa)} A^{(\kappa)\mu}\right), \\
\mathcal{L}_{int} =& - g_\sigma \bar{\Psi}\Phi_\sigma \Psi - g_\delta \bar{\Psi}\boldsymbol{\tau}\boldsymbol{\Phi}_\delta \Psi \\
& - g_\omega \bar{\Psi}\gamma_\mu A^{(\omega)\mu}\Psi - g_\rho \bar{\Psi}\gamma_\mu \boldsymbol{\tau}\mathbf{A}^{(\rho)\mu}\Psi \\
& - e\bar{\Psi}\gamma_\mu \frac{1}{2}(1+\tau_3)A^{(\gamma)\mu}\Psi,
\end{aligned}$$

with the field strength tensor: $F_{\mu\nu(\kappa)} = \partial_\mu A_{\nu(\kappa)} - \partial_\nu A_{\mu(\kappa)}.$

Density Dependent Relativistic Mean–Field Calculations

Realistic approach:
Dirac Brueckner Hartree–Fock for nuclear matter
E.N.E. van Dalen et al., EPJ A31, 29 (2007)
T. Klähn et al., PR C74, 035802 (2006)

$\Downarrow$

Adjustment of density dependent coupling functions to reproduce self–energy from DBHF results (LDA), and finite nuclei.

$\Downarrow$

Phenomenological approach:
Density Dependent Relativistic Mean Field (DDRMF) for nuclear matter, finite nuclei, and the *Wigner–Seitz Cell*.