4d Yang–Mills theory in terms of gauge invariant dual variables

a) duality transformation

b) gauge invariant variables

c) perturbation theory

d) “more general relativity”

e) the $\frac{11}{3}$

Speculations:

a) Gauge-invariant mass generation for dual gluons?

b) Renormalizable gravity? Grand Unification?

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The $4d$ YM partition function can be identically rewritten with the help of an additional Gaussian integration over dual field strength variables [Deser and Teitelboim (1976); Halpern (1977)]. This is called "the $1^{\rm st}$ order formalism".

$$Z_{\rm 4d} \;=\; \int DA_\mu \,\exp\int d^4x\,\left(-\frac{1}{2g_4^2}{\rm Tr}\;F_{\mu\nu}F_{\mu\nu}\right)$$

$$\left[\;\; F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu - i[A_\mu A_\nu]\;\;\right]$$

$$=\int DA_\mu\, DB_{\mu\nu}\,\exp\int d^4x\left(-\frac{g_4^2}{2}\,{\rm Tr}\;B_{\mu\nu}B_{\mu\nu}+\frac{i}{2}\,\epsilon^{\alpha\beta\mu\nu}\,{\rm Tr}\;B_{\alpha\beta}F_{\mu\nu}\right).$$

Both terms are invariant under (N^2-1)-function gauge transformation (in $SU(N)$)

$$\left\{\begin{array}{ccc}\delta A_\mu & = & [D_\mu\,\alpha],\\ \delta B_{\mu\nu} & = & [B_{\mu\nu}\,\alpha]\end{array}\right. ,\qquad D_\mu=\partial_\mu - iA_\mu,\qquad [D_\mu D_\nu]=-iF_{\mu\nu}.$$

Owing to the Bianchi identity, $\epsilon^{\mu\nu\rho\sigma}\,[D_\nu F_{\rho\sigma}]=0$, the second (mixed) term is **in addition** invariant under the $4(N^2-1)$-function 'dual' gauge transformation,

$$\left\{ \begin{array}{rcc} \delta A_\mu & = & 0, \\ \delta B_{\mu\nu} & = & [D_\mu \beta_\nu] - [D_\nu \beta_\mu]. \end{array} \right.$$

Taking a particular combination of the functions α, β_μ

$$\alpha = -v^\mu A_\mu, \qquad \beta_\mu = -v^\lambda B_{\lambda\mu},$$

leads to the 4-function transformation

$$\delta B_{\mu\nu} = B_{\lambda\nu}\,\partial_\mu v^\lambda + B_{\mu\lambda}\,\partial_\nu v^\lambda + \partial_\lambda B_{\mu\nu}\,v^\lambda,$$

being the known variation of a (covariant) tensor under infinitesimal general coordinate transformation, $x^\mu \to x^\mu + v^\mu(x)$, also called the diffeomorphism. Therefore, the 'mixed' term is diffeomorphism-invariant, and for the $SO(4)$ gauge group is known as **BF gravity**.

One can now perform the Gaussian integration over A_μ [Halpern (1977), Ganor and Sonnenschein (1996)]: the 'mixed' term must be invariant under $4(N^2-1)$ local transformations out of which 4 are diffeomorphisms.

“More general relativity”

In $4d$ the dual field strength is an antisymmetric tensor, not a vector, so to construct the metric tensor one has to be more industrious. We present the dual field strength in $SU(2)$:

$$(18-3) \quad B^a_{\mu\nu} = d^a_i\, T^i_{\mu\nu} = d^a_i\, \eta^i_{AB}\, e^A_\mu\, e^B_\nu, \qquad \eta^i_{AB} \text{ is the ‘t Hooft symbol}$$

$$(10) \quad g_{\mu\nu} = \frac{\epsilon^{\alpha\beta\rho\sigma}\ \epsilon_{abc}}{12\sqrt{g}} B^a_{\mu\alpha} B^b_{\rho\sigma} B^c_{\beta\nu} \quad \overset{d}{=} \quad e^A_\mu e^A_\nu,$$

$$(5) \quad h_{ij} = d^a_i\, d^a_j, \qquad \det h = 1.$$

There are 16 dof’s in the tetrad e^A_μ, however three rotations under one of the $SO(3)$ subgroups of the $SO(4)$ Lorentz group does not enter at all into $T^i_{\mu\nu}$, while rotations from the second $SO(3)$ can be absorbed into the rotations of d^a_i and hence into h_{ij}. Therefore, the antisymmetric tensor $T^i_{\mu\nu}$ carries 10 dof’s, as does $g_{\mu\nu}$. h_{ij} carries 5 dof’s. Hence, there are 10+5=15 gauge invariant variables, as it should be.

After integrating out A_μ from the 1^{st}-order partition function and expressing the result through T and h one obtains the $4d$ YM partition function in terms of gauge-invariant variables [D.D. and Petrov (2001)]

The saddle-point eqn. for $\bar{A}_\mu$:

$$\begin{aligned} 0 &= \epsilon^{\lambda\mu\alpha\beta}\, D^{ab}_\mu(\bar{A})\, B^b_{\alpha\beta} = \epsilon^{\lambda\mu\alpha\beta}\, D^{ab}_\mu(\bar{A})\, \left(d^b_i\, T^i_{\alpha\beta}\right) \\ &= \epsilon^{\lambda\mu\alpha\beta} \left(\left(D^{ab}_\mu\, d^b_i\right) T^i_{\alpha\beta} + d^a_i \left(\partial_\mu T^i_{\alpha\beta}\right)\right) = d^a_j\, (\nabla_\mu)^j_i \left(T^i_{\alpha\beta}\, \epsilon^{\lambda\mu\alpha\beta}\right), \end{aligned}$$

$$D^{ab}_\mu\, d^b_i \overset{d}{=} d^a_j (\gamma_\mu)^j_i \qquad \text{is the "minor" Christoffel symbol}$$

$$(\nabla_\mu)^j_i \overset{d}{=} \delta^j_i \partial_\mu + (\gamma_\mu)^j_i \qquad \text{is the "minor" covariant derivative.}$$

The Christoffel symbol $(\gamma_\mu)^j_i$ can be found explicitly through T, h from the condition that $T^i_{\mu\nu}$ and h_{ij} are "covariantly constant":

$$\begin{aligned} 0 &= h_{ij;\,\mu} \overset{d}{=} \partial_\mu\, h_{ij} - (\gamma_\mu)^k_i\, h_{kj} - h_{ik} (\gamma_\mu)^k_j, \\ 0 &= T_{\kappa\lambda;\,\mu} + T_{\lambda\mu;\,\kappa} + T_{\mu\kappa;\,\lambda}. \end{aligned}$$

Explicitly,

$$\begin{aligned} \gamma^j_{\mu i} &= \tfrac{1}{2}\, h^{jn} (\partial_\mu h_{ni} + \epsilon_{nik}\, S^k_\mu), \\ S^k_\mu &= T^k_{\nu\beta} T^l_{\mu\alpha} g^{\alpha\beta} \left(h_{lm} \partial_\lambda T^{m\lambda\nu} + \frac{1}{2g}\, T^{m\lambda\nu} \partial_\lambda (g h_{lm}) \right). \end{aligned}$$

The commutator of covariant derivatives is the 'minor' Riemann tensor

$$\begin{aligned} R^{j}_{\ i\,\mu\nu} &\overset{d}{=} [\nabla_\mu\, \nabla_\nu]^j_i \\ &= \partial_\mu \gamma^j_{\nu i} - \partial_\nu \gamma^j_{\mu i} + \gamma^j_{\mu k}\gamma^k_{\nu i} - \gamma^j_{\nu k}\gamma^k_{\mu i}. \end{aligned}$$

To find the YM field strength at the saddle point $F_{\mu\nu}(\bar{A})$ consider the double commutator of the YM covariant derivatives,

$$[\mathcal{D}_\mu[\mathcal{D}_\nu d_i]] = [\mathcal{D}_\mu,\ \gamma^j_{\nu i} d_j] = d_j(\partial_\mu \gamma^j_{\nu i} + \gamma^j_{\mu k}\gamma^k_{\nu i}),$$

and subtract the same commutator with $(\mu \leftrightarrow \nu)$ interchanged:

$$\begin{aligned} &[\mathcal{D}_\mu[\mathcal{D}_\nu d_i]] - [\mathcal{D}_\nu[\mathcal{D}_\mu d_i]] = -[d_i[\mathcal{D}_\mu \mathcal{D}_\nu]] \\ &= i[d_i\, F_{\mu\nu}(\bar{A})] = d_j\, R^{j}_{\ i\,\mu\nu}. \end{aligned}$$

The YM field strength at the saddle point $\bar{F}_{\mu\nu}$ is proportional to the Riemann tensor:

$$F^a_{\mu\nu}(\bar{A}) = \tfrac{1}{2}\,\epsilon^{abc}\, d^{bi}\, d^c_j\, R^{j}_{\ i\,\mu\nu}\,.$$

The 4d YM partition function is, in terms of gauge invariant variables h_{ij} and $T^i_{\mu\nu}$:

$$\mathcal{Z}^{4\mathrm{d}} = \int DhDT\, e^{\,Æ+\mathrm{BF}},$$

$$Æ = -\frac{g_4^2}{4}\int d^4x\, T^i_{\mu\nu}\, h_{ij}\, T^j_{\mu\nu}, \qquad \text{(the “æther” term)}$$

$$\mathrm{BF} = \frac{i}{4}\int d^4x\,\sqrt{g}\,R^j_{\ i\,\mu\nu}\,h^{im}\,T^{k\,\mu\nu}\,\epsilon_{jkm} \qquad \text{(the “Einstein – Hilbert” term).}$$

BF is invariant under a 12-function **local** transformation, induced by the invariance under dual gauge transformations. Four of these local transformations are diffeomorphisms.

Another possible term invariant under 12-function transformations, is the generalization of the cosmological term

$$\mathrm{CT} = \int d^4x\,\mathrm{Tr}\ h\,\sqrt{g}$$

In the particular case when $h_{ij} = \delta_{ij}$ the BF action reduces (!) to the standard Einstein–Hilbert action $\sqrt{g}R$, where R is the usual scalar curvature made of $g_{\mu\nu}$ whereas the “cosmological term” becomes the standard one.

Gauge-invariant perturbation theory

0^{th} order in the coupling g_4^2 requests the curvature of the dual space to be zero, $R^j_{\ i\,\mu\nu}=0$
It implies that the 'minor' Christoffel symbol $\gamma^i_{\mu j}$ is a "pure gauge",

$$\gamma^i_{\mu j}=\left(O^{-1}\right)^i_k\partial_\mu O^k_j,\qquad \det O\neq 0.$$

Indeed, in this case the Riemann tensor is zero:

$$\begin{aligned}(\nabla_\mu)^i_j\,c^j &= \partial_\mu c^i+\gamma^i_{\mu j}c^j=\left(O^{-1}\right)^i_k\partial_\mu\left(O^k_jc^j\right);\\ R^i_{\ j\,\mu\nu}c^j &= \left[(\nabla_\mu)^i_k(\nabla_\nu)^k_j-(\mu\leftrightarrow\nu)\right]c^j\\ &= \left(O^{-1}\right)^i_l\partial_\mu\partial_\nu\left(O^l_jc^j\right)-(\mu\leftrightarrow\nu)=0\end{aligned}$$

for any vector c^j, therefore, $R^i_{\ j\,\mu\nu}=0$.

We get

$$0=h_{ik;\,\mu}=\partial_\mu\left[\left(O^{-1}\right)^p_l h_{pq}\left(O^{-1}\right)^q_m\right]\,O^m_iO^l_k$$

meaning that $h_{pq} = O^i_p O^j_q D_{ij}$ where D_{ij} is a constant matrix. Next, we obtain

$$\partial_\kappa \left(O^k_j T^j_{\lambda\mu} \right) + \partial_\lambda \left(O^k_j T^j_{\mu\kappa} \right) + \partial_\mu \left(O^k_j T^j_{\kappa\lambda} \right) = 0,$$

whose general solution is $O^k_j T^j_{\kappa\lambda} = \partial_\kappa b^k_\lambda - \partial_\lambda b^k_\kappa$ where b^k_λ may be called the dual gauge potential. The 'æther' term in the action (Æ) is then

$$\begin{aligned} B^a_{\mu\nu} B^a_{\mu\nu} &= h_{pq} T^p_{\mu\nu} T^q_{\mu\nu} = D_{ij} (O^i_p T^p_{\mu\nu})(O^j_q T^q_{\mu\nu}) \\ &= D_{ij} \left(\partial_\mu b^i_\nu - \partial_\nu b^i_\mu \right) \left(\partial_\mu b^j_\nu - \partial_\nu b^j_\mu \right). \end{aligned}$$

D_{ij} is a constant matrix and can be set to be δ_{ij} by a linear transformation of the three vector fields b^i_μ; therefore, The 'æther' term becomes then

$$\text{Æ} \to \int d^4x (\partial_\mu b^i_\nu - \partial_\nu b^i_\mu)^2, \qquad i = 1, 2, 3,$$

describing three free massless dual gluons, as it should be.

12-function local transformations

The BF part of the YM action is invariant under transformations with 12 functions $z^i_\mu(x)$. This broad invariance follows from the invariance of the BF action under dual gauge transformations. In the infinitesimal form the 12 transformations read

$$\begin{aligned}
\delta p^i_{\mu\nu} &= (\nabla_\mu)^i_j\,\delta z^j_\nu - (\nabla_\nu)^i_j\,\delta z^j_\mu, \quad \delta Q^k_i = \frac{1}{4}\left(\delta^l_i\delta^k_m - \frac{1}{3}\delta^k_i\delta^l_m\right)\delta p^l_{\alpha\beta}\,T^{m\,\alpha\beta}\,,\\
\delta T^k_{\mu\nu} &= \delta p^k_{\mu\nu} - \delta Q^k_i\,T^i_{\mu\nu}\,,\\
\delta h_{ij} &= \delta Q^k_i\,h_{kj} + h_{ik}\,\delta Q^k_j \qquad \delta Q^i_i = 0\\
\delta g_{\mu\nu} &= \frac{1}{2}\left(g_{\mu\beta}p^i_{\alpha\nu} + g_{\beta\nu}p^i_{\alpha\mu} - \frac{1}{3}g_{\mu\nu}p^i_{\alpha\beta}\right)T^{i\,\alpha\beta}\,.
\end{aligned}$$

Corollary

The generalization of the cosmological term $\mathrm{CT} = \int d^4x\,\sqrt{g}\,\mathrm{Tr}\;h$ is an invariant of the 12-function transformation, too!

Question

What are other 12-function invariants, if any?

Variations above are in fact variational equations integrating which one obtains the transformations of $T, h, g_{\mu\nu}, R^{j}_{i\,\mu\nu}\ldots$ for *finite* functions $z^i_\mu(x)$.

[Example: the variational equation for the diffeomorphisms is

$$\delta g_{\mu\nu}(x,v(x)) = g_{\lambda\nu}\partial_\mu \delta v^\lambda + g_{\mu\lambda}\partial_\nu \delta v^\lambda + \partial_\lambda g_{\mu\nu}\,\delta v^\lambda$$

whose solution is $g_{\mu\nu}(x,v(x)) = g_{\alpha\beta}(w)\,\partial_\mu w^\alpha\,\partial_\nu w^\beta,\ w^\mu(x) = x^\mu + v^\mu(x)$.]

In the particular case when $z^i_\mu(x) = -T^i_{\mu\nu}\,v^\nu$ the 12 transformations reduce to 4 **general coordinate transformations** (or diffeomorphisms) parametrized by the vector v^ν:

$$\begin{array}{lcll} h_{ij}(x) & \longrightarrow & h_{ij}(w), & w^\mu(x) = x^\mu + v^\mu(x), \\ g_{\mu\nu}(x) & \longrightarrow & g_{\alpha\beta}(w)\,\partial_\mu w^\alpha\,\partial_\nu w^\beta\,, & \ldots \end{array}$$

Big questions

We have actually found a new gauge principle: an invariance under $4\cdot\dim(G)$ **local** transformations. Diffeomorphisms are their small subset.

- How to write those transformations in a finite form?
- What is their geometrical meaning?
- Can this symmetry be broken spontaneously (like in the Higgs effect)?

Gauge invariant saddle point: the de Sitter space

We look for the saddle point of the full action, “æther” + “Einstein-Hilbert”, in the simplest form:

$$h_{ij} = \delta_{ij}, \qquad g_{\mu\nu} = \Phi(x^2)\delta_{\mu\nu}\,.$$

Φ is readily found from the saddle-point equation:

$$\Phi(x) = \frac{4i}{g_4^2}\,\frac{\rho^2}{(x^2+\rho^2)^2} \quad\Longrightarrow\quad R = ig_4^2 \quad\Longrightarrow\quad \text{Action} \;=\frac{8\pi^2}{g_4^2}.$$

Actually it is the BPST instanton in disguise. The usual YM instanton corresponds to the dual space of constant imaginary curvature S^4 !

Small fluctuations about the saddle point

The BF action is invariant under the 12-function transformations $z_\mu^i(x)$ but the Æterm is not. In the quadratic order in $z_\mu^i(x)$ it transforms to

$$Æ + BF \;\rightarrow -\frac{8\pi^2}{g_4^2} + \tfrac{1}{2}\int d^4x\; z_\mu^i\, h_{ik}\left(\delta_{\mu\nu}(\nabla^2)_j^k - (\nabla_\mu\nabla_\nu)_j^k + 2R^k_{j\;\mu\nu}\right) z_\nu^j + O(z^3).$$

The determinant of the quadratic operator becomes, after proper normalization to flat space and dimensional regularization,

$$\log\ \mathrm{det}^{-\frac{1}{2}}_{\mathrm{norm,reg}}(-\delta_{\mu\nu}\nabla^2+\nabla_\mu\nabla_\nu-2R_{\mu\nu})=\frac{11}{3}\,\frac{(M^2)^\epsilon}{\epsilon},$$

where "11/3" is the correct 1-loop renormalization of the $SU(2)$ YM theory!

Large fluctuations about the saddle point

Let us freeze all z^i_μ's except four general coordinate transformations (diffeomorphisms) $w^\alpha(x)$. The fluctuation of the æther term in the direction of the diffeomorphisms is known exactly in all orders in $w^\alpha(x)$:

$$4d:\qquad Æ=\frac{4}{g_4^2}\int d^4x\,\rho^4\,\frac{\partial_\mu w^\alpha\partial_\mu w^\alpha\partial_\nu w^\beta\partial_\nu w^\beta-\partial_\mu w^\alpha\partial_\nu w^\alpha\partial_\mu w^\beta\partial_\nu w^\beta}{(w^2+\rho^2)^4}$$

$$=\qquad\frac{1}{4g_4^2}\int d^4x\,\left(\partial_\mu n^A\partial_\mu n^A\partial_\nu n^B\partial_\nu n^B-\partial_\mu n^A\partial_\nu n^A\partial_\mu n^B\partial_\nu n^B\right),$$

$$n^{1,2,3,4}=\frac{2w^{1,2,3,4}\,\rho}{\rho^2+w^2},\quad n^5=\frac{\rho^2-w^2}{\rho^2+w^2},\qquad\sum_{A=1}^{5}n^An^A=1.$$

At $w^\alpha(x)=x^\alpha$ it becomes the standard instanton: $12\rho^4/(x^2+\rho^2)^4$.
At $\mathbf{w}(x)=\mathbf{x}$, $w^4=-x^4$ it becomes the standard anti-instanton. In general, $w^\alpha(x)$

corresponding to winding number N_w gives a configuration with topological charge N_w.

It is similar to the $2d$ $O(3)$-sigma model

$$2d \quad : \qquad \frac{1}{f}\int d^2x\,\rho^2\,\frac{\partial_\mu w^\alpha \partial_\mu w^\alpha}{(w^2+\rho^2)^2} = \frac{1}{4f}\int d^2x\,\partial_\mu n^A \partial_\mu n^A,$$

$$n^{1,2} = \frac{2w^{1,2}\,\rho}{\rho^2+w^2}, \quad n^3 = \frac{\rho^2-w^2}{\rho^2+w^2}, \qquad \sum_{A=1}^{3} n^A n^A = 1.$$

The fate of the σ-model in the non-perturbative regime is the spontaneous generation of mass for **three** fields n^A. For a general $O(n)$ σ-model one writes (A=1...n)

$$\int D\lambda\,Dn^A\,\exp\left[-\frac{1}{4f}\int d^2x\,\left(\partial_\mu n^A \partial_\mu n^A + \lambda(n^A n^A - 1)\right)\right].$$

The average Lagrange multiplier λ is found from the saddle point, justified at large n:

$$0 = \frac{\partial}{\partial\lambda}\left[-\frac{n}{2}\int \frac{d^2p}{(2\pi)^2}\,\ln(p^2+\lambda) + \frac{\lambda}{4f}\right] \quad \Longrightarrow \quad \frac{2\pi}{f} = (n+\mathcal{O}(1))\,\ln\frac{M^2}{\lambda}.$$

The exact asymptotic freedom coefficient is in fact $n-2$; $\bar{\lambda} = M^2\exp\left(-\frac{2\pi}{(n-2)f}\right)$.

A mass gap for dual gluons?

Similarly, in the $4d$ lagrangian for the diffeomorphisms of the æther term, five fields $n^A(x)$ generate a mass; in the limit $5 = n \to \infty$ it can be seen from the Lagrange multiplier developing a nonzero v.e.v. We find the mass m of the n^A field

$$\frac{8\pi^2}{g_4^2} = \frac{2n}{3}\,\ln\frac{M^2}{m^2}\bigg|_{n=5} = \frac{10}{3}\,\ln\frac{M^2}{m^2}\,, \qquad \text{instead of } \frac{11}{3}\,, \qquad \text{a } 10\% \text{ error.}$$

$$m^2 = M^2\,\exp\left(-\frac{8\pi^2}{\frac{10}{3}g_4^2}\right)\,.$$

We know already that the difference is corrected by taking into account the fluctuations in the non-diffeomorphism directions of the general z^i_μ.

n^A spontaneously obtaining a mass means **dual gluons get a mass** (through dimensional transmutation), which is usually considered to be necessary for the confinement.

Renormalizable gravity?

$$\mathrm{BF} \;=\; \frac{i}{4}\int d^4x\,\sqrt{g}\,R^j_{\;i\,\mu\nu}\,h^{im}\,T^{k\,\mu\nu}\,\epsilon_{jkm} \qquad \left(= \frac{i}{2}\int d^4x\,\epsilon^{\alpha\beta\mu\nu}\,\mathrm{Tr}\;B_{\alpha\beta}F_{\mu\nu}\right).$$

This is an analogue of the Einstein–Hilbert action, but thanks to additional 5 scalar fields h_{ij} it is invariant under 12-function **local** transformations, **four of which are general coordinate transformations or diffeomorphisms.**

Another possible term invariant under 12-function transformations, is the generalization of the cosmological term

$$\mathrm{CT} \;=\Lambda\int d^4x\,\sqrt{g}\;\mathrm{Tr}\,h \qquad \left(=\Lambda\int d^4x\,\epsilon^{\kappa\lambda\mu\nu}\,\mathrm{Tr}\;B_{\kappa\lambda}B_{\mu\nu}\right).$$

Integrating out $B_{\mu\nu}$ (the integral is Gaussian) we get

$$\mathcal{Z}=\int DA^a_\mu\,\exp\left(i\int d^4x\,\epsilon^{\kappa\lambda\mu\nu}\,F^a_{\kappa\lambda}\left(\frac{1}{\Lambda}\right)F^a_{\mu\nu}\right)$$

This is the "θ-term", and clearly shows that the theory is renormalizable!

In the particular case when $h_{ij}=\delta_{ij}$ the BF action reduces (!) to the standard Einstein–Hilbert action $\sqrt{g}R$, where R is the usual scalar curvature made of $g_{\mu\nu}$, whereas the "cosmological term" becomes the standard one!

$$\frac{i}{4}\int d^4x\,\sqrt{g}\,R^j_{i\,\mu\nu}\,h^{im}\,T^{k\,\mu\nu}\,\epsilon_{jkm} \quad \overset{h_{ij}\to\delta_{ij}}{\longrightarrow} \quad \frac{i}{2}\int d^4x\,\sqrt{g}\,R$$

$$\Lambda\int d^4x\,\sqrt{g}\;\mathrm{Tr}\,h \quad \overset{h_{ij}\to\delta_{ij}}{\longrightarrow} \quad \Lambda\int d^4x\,\sqrt{g}.$$

How to "squeeze" 5 extra fields to be $h_{ij} = \delta_{ij}$? One way to do it – by integrating over 5 auxiliary fields λ^{ab}, $\lambda^{ab} = \lambda^{ba}$, $\lambda^{aa} = 0$:

$$\int D\lambda^{ab}\,\exp\left(i\int d^4x\,\epsilon^{\kappa\lambda\mu\nu}\,B^a_{\kappa\lambda}B^b_{\mu\nu}\,\lambda^{ab}\right) = \delta\left(h_{ij}-\delta_{ij}\right).$$

This action is also invariant under 12-function dual gauge transformations, provided the transformation of λ^{ab} is appropriately chosen.

One can make this symmetry breaking "soft" by adding a smearing

$$H\int d^4x\,\sqrt{g}\,\mathrm{Tr}\,\lambda^2\,.$$

Taking the (Gaussian) integral over $B_{\mu\nu}$ one can represent the standard quantum gravity as

$$\mathcal{Z} = \int DA_\mu^a \, D\lambda^{ab} \, \exp\left(i \int d^4x \, \epsilon^{\kappa\lambda\mu\nu} \, F_{\kappa\lambda}^a \left(\frac{1}{\Lambda + \lambda} \right)^{ab} F_{\mu\nu}^b \right)$$

Λ is the
cosmological constant

I think this theory is also renormalizable.

Some CONCLUSIONS

1. One can rewrite the quantum YM theory exactly in terms of local gauge-invariant variables. In $3d\ SU(2)$ these are the six external coordinates $X^{1,2,3,4,5,6}(x)$ describing the embedding of the curved dual space into flat space.

2. We have rewritten the $4d$ BF theory in a basis-independent way. The action is "$R\sqrt{g}$" (also "$\sqrt{g}$") but invariant under 12 local transformations, 4 of which are diffeomorphisms. The geometrical meaning of the other 8 is still unclear.

3. The physical meaning of the 12 functions: they describe dual gluons, in a gauge-invariant fashion. We reproduce the 11/3 of the YM theory from integrating over these 12 local transformations.

4. Plausibly, the non-perturbative YM vacuum corresponds to a dual space of **constant curvature**, and dual gluons spontaneously obtain a mass from the Lagrange multiplier (forcing the fields to lie on a sphere) getting a vacuum expectation value.

5. A global scenario: Start from a renormalizable (BF+CT+...) theory based on some large Lie group. Part of the symmetries are spontaneously broken by an "æther" term $\Longrightarrow$ this part is the Standard Model. Another part is spontaneously broken by $< h_{ij} >= \delta_{ij}$ $\Longrightarrow$ this part is the Standard Gravity. **Æ, BF** .